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N.N. Krasovskii Institute of Mathematics and Mechanics of the Ural Branch of the Russian Academy of Sciences
The Ural Mathematical Center

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Yekaterinburg – Online, Russia, August 24-30, 2020

Abstracts

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All the Timetable is scheduled for Yekaterinburg Time

(London Time =Yekaterinburg Time - 4hrs; CEST Time = Yekaterinburg Time - 3hrs; Moscow Time = Minsk Time = Yekaterinburg Time - 2hrs; Novosibirsk Time = Yekaterinburg Time + 2hrs; Beijing Time = Perth Time = Yekaterinburg Time + 3hrs; Tokyo Time = Yekaterinburg Time + 4hrs).

Monday, 24 August

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13:00 – 13:50 Arseny Shur: *Words separation and positive identities in symmetric groups*
14:30–15:30 Open Problems Session. Chair: Natalia Maslova
15:30 Official Closing of the Workshop

Plenary Talks

Latin cubes

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The Latin square is a familiar combinatorial object. If it has order n, it consists of an $n \times n$ square array of cells, with one letter in each cell, in such a way that each letter occurs exactly once in each row and column.

If the labels of the rows and columns are the same letters that occur in the array, then the Latin square becomes the Cayley table of a quasigroup. Cayley tables of groups are a special (and rather rare) case of this. There is a combinatorial condition called the *quadrangle criterion.* A Latin square is the Cayley a table of a group (possibly after suitable permutations of the labels and letters) if and only if it satisfies the quadrangle criterion: in this case, the group is determined up to isomorphism.

Another way of thinking about a Latin square is as a set of three partitions of the set of n^2 cells: the parts are rows, columns and letters respectively. Any two of these partitions, together with the two trivial partitions, give a Cartesian lattice of dimension two, which is sometimes called a *grid.*

Now let us increase the dimension and think about cubes. What should a Latin cube of order n be? Several different definitions have been given, all involving a three-dimensional $n \times n \times n$ array of cells. Some definitions have n letters; some have n^2 letters; some simply have a subset of the cells.

I will use the definition that has n^2 letters, each occurring once in every two-dimensional slice of the cube. Such a cube has four natural partitions into n^2 parts of size n. The parts of one partition are the one-dimensional slices (or *lines*) parallel to the x-axis. Similar partitions are defined by the y-axis and the z-axis. The parts of the fourth partition are the letters.

Such a Latin cube is called *regular* if the sets of letters in any two parallel lines of the cube are either the same or disjoint.

I will describe some very recent work (joint with Peter Cameron, Cheryl Praeger and Csaba Schneider) that shows that the following three conditions on a Latin cube (as defined above) are equivalent.

- The Latin cube is regular.
- Every set of three of the four partitions described above form the minimal non-trivial partitions in a Cartesian lattice of dimension three.
- There is a group T of order n, determined uniquely up to isomorphism, such that the cells of the cube can be identified with the elements of T^3 in such a way that the letter partition is the partition of T^3 into the right cosets of its *diagonal subgroup*, which is $\{(t, t, t) : t \in T\}$.

Amazingly, this result does not require n to be finite.

This result is the foundation step of a proof by induction of a similar result for all dimensions bigger than two.

From de Bruijn graphs to automorphisms of the shift

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This is joint work with Collin Bleak and Feyishayo Olukoya

A (finite-state, deterministic) automaton is a machine with a finite set of internal states; when it reads a symbol from a given input alphabet it changes its state in a deterministic way. An automaton can be represented by an edge-labelled directed graph whose vertices are the states, edge-labels corresponding to the symbols in the alphabet, so that there is an edge with label a from s to t if reading symbol a causes the machine to change from state s to state t. If the machine reads the symbols of a word in order, it undergoes the concatenation of the corresponding transitions.

An automaton is *synchronizing* if there is a word w (called a *reset word*) such that reading w brings it to a known state, independent of its start state. However, we need a stronger concept: we call an automaton *strongly synchronizing* if there is a positive integer n such that every word of length n is a reset word. The smallest such n is called the *synchronization length* of the automaton.

The *de Bruijn graph* $G(n, A)$ of word length n over an alphabet A has as vertex set the set A^n of words of length n over A, with an edge labelled a from $b_1b_2 \cdots b_n$ to $b_2 \cdots b_n a$. It is a strongly synchronizing automaton with synchronization length n. Indeed, any strongly synchronizing automaton with synchronization length n over A is a *folding* of the de Bruijn graph $G(n, A)$, the quotient by an equivalence relation respecting the action of the automaton.

How many strongly synchronizing automata (i.e. foldings of the de Bruijn graph) with synchronization length at most n over an alphabet of size q are there? This is an interesting but difficult problem. For $n = 1$, the answer is the Bell number $B(q)$, the number of partitions of the alphabet. We have a formula for the case $n = 2$, but have been unable to solve the problem in general.

A *transducer* is an automaton which can write as well as read symbols from the alphabet A. We always assume that a transducer "eventually writes something", that is, if it reads a word which returns it to the starting state, it must write a non-empty word. Now we can regard a transducer with this property as defining a function from the set A^ω of all infinite sequences of elements of A to itself. We can identify A^ω with the Cantor set (giving it the Tychonoff product topology given by the discrete topology on A); maps given by transducers are continuous.

We are particularly interested in automata for which this function is invertible, and the inverse is also computed by a transducer. These transformations form a group of permutations of A^ω, which are homeomorphisms of Cantor space, forming the *rational group* $\mathcal{R}_q$ of Grigorchuk, Nekrashevych and Suschanskiĭ.

The starting point of our investigation concerned the outer automorphism groups of the *Higman–Thompson groups*, the first infinite family of finitely presented infinite simple groups to be constructed (the first such group was found by Richard Thompson, and the construction generalised by Graham Higman). The crucial observation is that, roughly, the outer automorphisms are induced by transducers which are *bisynchronizing*, that is, both the map and its inverse are given by transducers whose underlying automata are strongly synchronizing.

In symbolic dynamics, it is common to consider the set $A^\mathbb{Z}$ of two-way infinite sequences of symbols from a finite alphabet A. The *shift map* acts on $A^\mathbb{Z}$ by moving each symbol one place left. An *automorphism* of the shift is a permutation of $A^\mathbb{Z}$ which commutes with the shift. One difficulty in studying such automorphisms is in indexing the coordinates in a suitable way. Our recent work gives a representation of automorphisms of the shift by bisynchronizing transducers with added structure called an *annotation* which allows us to handle this problem.

In the talk I will attempt to give some account of all these matters.

On Thompson's conjecture for finite simple groups

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Consider a finite group G. For $g \in G$, denote by g^G the conjugacy class of G containing g, and by $|g^G|$ the size of g^G. Put $N(G) = \{|g^G| \mid g \in G\} \setminus \{1\}$. In 1987, John Thompson posed the following conjecture concerning $N(G)$.

Thompson's Conjecture (see Kourovka Notebook, Question 12.38). *If L is a finite simple non-abelian group, G is a finite group with trivial center, and $N(G) = N(L)$, then $G \simeq L$.*

Note that the same question can be formulated for an arbitrary finite group with the trivial center.

We say that the group L is recognizable by the set of conjugacy class sizes among finite groups with trivial center (briefly recognizable) if the equality $N(L) = N(G)$, where G is a finite group with a trivial center, implies the isomorphism $L \simeq G$. Since $N(L) = N(L \times Z(L))$, the condition $Z(L) = 1$ is essential. However, it is easy to show that S_3 is recognizable. Thus, the condition of solvability is not a necessary condition for the recognizability. As an example of a non-recognizable group, we can take a Frobenius group of order 18. There exist two non-isomorphic Frobenius groups of order 18 with the same sets of conjugacy class sizes. G. Navarro found two finite groups G and H with trivial center such that $N(G) = N(H)$, G is solvable and H is non-solvable. It is open the question on the existence of a finite group L such that there exists an infinite set of pairwise non-isomorphic finite groups $\{G_i, i \in \mathbb{N}\}$ with trivial center such that $N(L) = N(G_i)$. The set $N(L)$ is closely connected with the order of the group L, and sometimes precisely determines it, which essentially limits the possibilities for constructing an infinite series of groups with a trivial center and the same set of conjugacy classes sizes.

We managed to prove the validity of Thompson's conjecture and to prove the recognizability of some non-simple groups.

The Weisfeiler–Leman stabilization revisited from the viewpoint of Terwilliger algebras

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Given a finite graph Γ, a coherent algebra is defined as the smalleast algebra that contains the adjacency matrix of Γ and is closed under both of the ordinary and the Hadamard products of matrices. A coherent configuration arises from the coherent algebra as its combinatorial realization. In this way, a coherent configuration is attached to a finite graph Γ and we call it the *coherent closure* of Γ. The Weisfeiler-Leman stabilization is the algorithm to derive the coherent closure of Γ.

What is important about the WL stabilization is the fact that the automorphism group of Γ coinsides with that of the coherent closure of Γ. In fact, the WL stabilization was proposed in order to investigate graph automorphisms. Historically speaking, the WL stabilizaiton appeared in the late 1960's in the then Soviet Union and reached the concept of coherent configurations. Note that the theory by D. G. Higman of coherent confirurations was published in the early 1970's. Along with the tragedy of Weisfeiler, the details are known not much to us, perhaps except for Russian contemporary specialists in his field.

Given a finite connected graph Γ and a fixed vertex of Γ, we can think of something like the coherent closure. It was defined by Paul Terwilliger in his unpublished lecture notes and we call it the Terwilliger algebra. So the Terwilliger algebra is closely related to the stabilizer of the fixed vertex in the automorphism group of Γ. Originally, the Terwilliger algebra was defined and deeply studied for an association scheme, which falls into a subclass of coherent configurations, by Paul Terwilliger in his monumental papers: The Subconstituent Algebra of an Association scheme I, II, III, J. Algebraic Combinatorics 1(1992), 363-388, 2(1993), 73-103, 177-210. In this talk, we first follow Terwilliger's lecture notes and consider the Terwilliger algebra of a finite connected graph. Then confining our targets to trees and distance-regular graphs, we revisit the WL stabilization from the viewpoint of representations of Terwilliger algebras.

Densely embedded subgraphs in locally projective graphs

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I will discuss some aspects of the classification of tilde and Petersen geometries accomplishes by Sergey Shpectorov and me some 20 years ago and put it in a more general setting.

On strongly Deza graphs

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This is joint work with Elena Konstantinova and Leonid Shalaginov

A *Deza graph* with parameters (n, k, b, a) is a k-regular graph on n vertices such that any two vertices have b or a common neighbours, $b \geqslant a$ [2].

The *children* G_A and G_B of a Deza graph G are defined on the vertex set of G such that every two distinct vertices are adjacent in G_A or G_B if and only if they have a or b common neighbours, respectively.

We call a Deza graph with strongly regular children as a *strongly Deza graph*. Obviously, strongly Deza graphs generalize divisible design graphs [3]. The following theorem is based on spectral relationships between Deza graphs and their children found in [1].

Theorem 1. *A Deza graph G is a strongly Deza graph if and only if G has at most three distinct absolute values of its eigenvalues.*

In our work we study properties and give some constructions of strongly Deza graphs [4].

Acknowledgement. The work is supported by Mathematical Center in Akademgorodok, the agreement with Ministry of Science and High Education of the Russian Federation number 075-15-2019-1613.

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[1] S. Akbari, A. H. Ghodrati, M. A. Hosseinzadeh, V. V. Kabanov, E. V. Konstantinova, L. V. Shalaginov, Spectra of Deza graphs *Linear Multilinear Algebra.* DOI: 10.1080/03081087.2020.1723472

[2] M. Erickson, S. Fernando, W.H. Haemers, D. Hardy, J. Hemmeter, Deza graphs: A generalization of strongly regular graphs *J. Comb. Des.* **7** (1999) 359–405.

[3] W. H. Haemers, H. Kharaghani, M. Meulenberg, Divisible design graphs *J. Comb. Theory, Ser. A* **118** (2011) 978–992. https://doi.org/10.1016/j.jcta.2010.10.003

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Efficient approximation of vehicle routing problems in metrics of a fixed doubling dimension

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Most of the valuable problems in combinatorial optimization including the classic Traveling Salesman Problem (TSP) and Vehicle Routing Problem (VRP) appear to be quite similar with respect to their computational complexity and approximability by algorithms with theoretical performance guarantees. Indeed, all these problems are strongly NP-hard and remain intractable even on the Euclidean plane. Although, in general, all of them are hard to approximate in polynomial time and APX-complete in arbitrary metric, their geometric settings admit quasi-polynomial or even polynomial-time approximation schemes, i.e. all of them can be approximated efficiently within any given accuracy. Recent results in the analysis of finite metric spaces shed a light on the design of approximation schemes for a wide family of metric settings of these problems. In this presentation, we try to illustrate some of such approaches on the example of the Capacitated Vehicle Routing Problem formulated in a metric space of an arbitrary fixed doubling dimension.

Dual Seidel switchings and integral graphs

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In this talk recent results on integral graphs obtained by dual Seidel switching and generalised dual Seidel switching are discussed. The talk is based on joint papers with S. V. Goryainov, H. Li, D. Zhao [1] and V. V. Kabanov, L. V. Shalaginov [2].

A graph is *integral* if all eigenvalues of its adjacency matrix are integers. Dual Seidel switching is a graph operation which can change the graph, however it does not change its bipartite double, and because of this, the operation leaves the squares of the eigenvalues invariant. Thus, if a graph is integral then it is still integral after dual Seidel switching. New infinite families of integral graphs obtained by dual Seidel switching are given; in particular, new 4-regular integral graphs are presented [1].

In [2] a generalisation of the dual Seidel switching is used to have some constructions of Deza graphs with strongly regular children. A Deza graph with strongly regular children is called a *strongly Deza graph*. We discuss integral strongly Deza graphs. In particular, for a singular strongly Deza graph the following result holds [3]. A graph is said to be *singular* if and only if zero is its eigenvalue.

Theorem. *Any singular strongly Deza graph is an integral graph with four distinct eigenvalues.*

Acknowledgement. The work is supported by Mathematical Center in Akademgorodok, the agreement with Ministry of Science and High Education of the Russian Federation number 075-15-2019-1613.

References

[1] S. V. Goryainov, E. V. Konstantinova, H. Li, D. Zhao, On integral graphs obtained by dual Seidel switching *Linear Algebra and its Applications* 604 (2020) 476–489.

[2] V. V. Kabanov, E. V. Konstantinova, L. V. Shalaginov, Generalised dual Seidel switching and Deza graphs with strongly regular children, arXiv:2005.13305v2 [math.CO]

[3] S. Akbari, M. A. Hosseinzadeh, V. V. Kabanov, E. V. Konstantinova, L. V. Shalaginov, Spectra of strongly Deza graphs, *in progress.*

On the parameters of unrestricted completely regular codes

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We discuss constructions of completely regular codes and proofs of their nonexistence for given parameters, mainly focusing on unrestricted (not necessarily linear) binary completely regular codes with covering radius at least 2.

On second maximal subgroups of finite groups

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1. The influence of generalized cover-avoidance property of subgroups on the construction of solvable groups and p-solvable groups.

A subgroup A of a finite group G is called a CAP-subgroup of G if for every chief factor H/K of G, we have $H \cap A = K \cap A$ or $HA = KA$.

Based on classifications of second maximal subgroups by Flavell, we investigate the structure of solvable groups by strong maximal subgroups having the cover-avoidance property or having the semi-cover-avoidance property. We choose the appropriate set of some second maximal subgroups with cover-avoidance property, obtain a necessary and sufficient condition to describe solvable groups.

Through further research, we obtain some characterizations for a solvable group (or a p-solvable group) based on the assumption that some maximal subgroups or second maximal subgroups have the cover and avoidance properties(or semi-p-cover and avoidance properties).

2. The influence of generalized cover-avoidance property of subgroups on the construction of generalized p-solvable groups.

Corresponding to the classes of p-solvable groups, we define generalized p-solvable groups S_p^* containing every group G whose every chief factor H/K satisfies one of the following conditions: (1) H/K is a p-group; (2) H/K is a $p^{'}$-group; (3) $|H/K|_p = p$ where H/K is nonsolvable.

In view of the definition and characteristics of generalized p-solvable groups and cover and avoidance properties, we introduced the notion of generalized p-solvable cover-avoidance property. Let A be a subgroup of G and H/K be a pd-chief factor of G. We will say that A have generalized p-solvable cover-avoidance property, if either $AH = AK$ or $|A \cap H : A \cap K|_p \leq p$.

In this talk, we introduce some characterizations for generalized p-solvable groups based on the assumption that some maximal subgroups or second maximal subgroups have generalized p-solvable cover-avoidance property.

Subgroups like minimal parabolic subgroups

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Let p be a prime. Suppose that $\mathcal{P}$ is the set of minimal parabolic subgroups of a simple group G of Lie type defined in characteristic p containing a fixed Sylow p-subgroup. For $R \in \mathcal{P}$, set $L = \langle \mathcal{P} \setminus \{R\} \rangle$. Then it is well known that $O_p(L) \not\leq O_p(R)$ and $L \neq G$.

More generally, a subgroup P of a group G is called p-minimal if P contains a Sylow p-subgroup S of G and S is contained in a unique maximal subgroup of P. Let $\mathcal{P}(S)$ be the set of p-minimal subgroups containing S. We say that $R \in \mathcal{P}(S)$ is isolated provides $L = \langle \mathcal{P} \setminus \{R\}, N_G(S) \rangle \neq G$ and $O_p(L) \not\leq O_p(R)$. We ask, what can be said about groups with an isolated p-minimal subgroup?

On the Weisfeiler–Leman dimension of Paley graphs

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It is proved that with finitely many possible exceptions, each cyclotomic scheme over a finite field is determined up to isomorphism by the tensor of 2-dimensional intersection numbers; for infinitely many schemes, this result cannot be improved. As a consequence, the Weisfeiler–Leman dimension of a Paley graph or tournament is at most 3 with possible exception of several small graphs.

Totally 2-closed groups

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This is joint work with Majid Arezoomand, Mohammadali Iranmanesh, and Gareth Tracey

A group G is said to be totally 2-closed if in each of its faithful permutation representations, say on a set X, X is the largest permutation group on X which leaves invariant each of the G-orbits for the induced action on ordered pairs from X.

This notion of 2-closure of a permutation group was introduced by Wielandt [1] in 1969. The stronger concept of total 2-closure, suggested by Holt, is independent of the permutation representation of the group. It was first explored by Abdollahi and Arezoomand [2] who showed that a finite nilpotent group is totally 2-closed if and only if it is either cyclic, or a direct product of a generalised quaternion group and a cyclic group of odd order. A more general study of total 2-closure was undertaken by Abdollahi, Arezoomand and Tracey in [3]. They showed in particular that among the finite soluble groups, the only totally 2-closed ones are nilpotent [3, Theorem B]. In addition they showed that the Fitting subgroup of a finite totally 2-closed group is itself totally 2-closed [3, Theorem A]. At the time of their writing the paper [3], no examples of insoluble totally 2-closed groups were known. They studied the structure of an insoluble example G of smallest order, showing that G modulo its cyclic centre has a unique minimal normal subgroup which is nonabelian.

Thus we set out to find some finite insoluble totally 2-closed groups G. We focused on the case where the Fitting subgroup is trivial, and showed first that such a group is a direct product of pairwise non-isomorphic nonabelian simple groups.

In the light of this result, we decided that our first challenge should be to find all totally 2-closed finite simple groups. The 2-closure condition seemed so strong that we initially believed that we would find no examples. Surprisingly (to us) such groups do indeed exist, though we still believe they are rare. We are still in the process of writing up our work, but we can report that, out of the 26 sporadic simple groups, exactly SIX of them are totally 2-closed, namely the Janko groups J_1, J_3 and J_4, together with Ly, Th and the Monster M. Along the way we developed several tools for studying finite totally 2-closed groups.

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Reduction theorems for relatively maximal subgroups

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Since its inception in the papers by É. Galois and C. Jordan, group theory has had the following as one of its central problems. Given a group G, find its subgroups possessing a specific property or, equivalently, belonging to a specific class $\mathfrak{X}$ of groups (for example, solvable, nilpotent, abelian, p-groups, etc.). If $\mathfrak{X}$ has good properties resembling those of solvable groups then to solve the general problem it suffices to know the so-called *maximal $\mathfrak{X}$-subgroups* (or *$\mathfrak{X}$-maximal subgroups*), i. e. the maximal by inclusion subgroup belonging to $\mathfrak{X}$.

Let $\mathfrak{X}$ be a non-empty class of finite groups closed under taking subgroups, homomorphic images and extensions. One of the main difficulties in studying $\mathfrak{X}$-maximal subgroups is that they behave irregularly under homomorphisms: if $\mathfrak{X}$ differs from the classes of all finite groups, the groups of order 1, and the classes of all p-groups where p is a prime, then arbitrary finite group G in the image of an epimorphism $G^* \to G$ from a finite group G^* such that every (not only maximal) $\mathfrak{X}$-subgroup of G is the image of an $\mathfrak{X}$-maximal subgroup of G^*. On the other hand, if the kernel of a homomorphism is an $\mathfrak{X}$-group or does not contain nontrivial $\mathfrak{X}$-subgroups then the image of each $\mathfrak{X}$-maximal subgroup is an $\mathfrak{X}$-maximal subgroup in the image of the homomorphism and, moreover, there is a natural bijection between the conjugacy classes of $\mathfrak{X}$-maximal subgroups in the domain and the image of the homomorphism. The main goal of the reported study is to describe all homomorphisms under which the $\mathfrak{X}$-maximal subgroups "behave well".

Denote by $\mathrm{k}_{\mathfrak{X}}(G)$ the number of conjugacy classes of $\mathfrak{X}$-maximal subgroups of a finite group G.

Let N be a normal subgroup of a finite group G. We say that *the reduction $\mathfrak{X}$-theorem holds for the pair* (G, N) if $\mathrm{k}_{\mathfrak{X}}(G) = \mathrm{k}_{\mathfrak{X}}(G/N)$. It is easy to show that the reduction $\mathfrak{X}$-theorem (G, N) implies that if H is an $\mathfrak{X}$-maximal subgroup of G then HN/N is an $\mathfrak{X}$-maximal subgroup of G/N and the map $H \mapsto HN/N$ induces a natural bijection between the conjugacy classes of $\mathfrak{X}$-maximal subgroups in G and G/N.

We say that *the reduction $\mathfrak{X}$-theorem holds for a finite group A* if it holds for every pair (G, N) such that N is a normal subgroup of G and N is isomorphic to A.

Theorem 1. *Let $\mathfrak{X}$ be a non-empty class of finite groups closed under taking subgroups, homomorphic images and extensions. For a finite group G, the following statements are equivalent:*

(i) *The reduction $\mathfrak{X}$-theorem hold for G;*

(ii) *The $\mathfrak{X}$-maximal subgroups of G are conjugate;*

(iii) *Every composition factor of G belongs to an explicitly given list.*

Theorem 2. *Let $\mathfrak{X}$ be a non-empty class of finite groups closed under taking subgroups, homomorphic images and extensions and let N be a normal subgroup of a finite group G. The following statements are equivalent:*

(i) *The reduction $\mathfrak{X}$-theorem hold for the pair (G, N);*

(ii) *The reduction $\mathfrak{X}$-theorem hold for the group N.*

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On the widths of finite groups

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Let G be a finite group and $e(G)$ (spectrum) the set of element orders in G. We call $w_o(G) = |(G)|$ the width of order of G and $w_s(G) = max\{|\pi(k)||k \in \pi_e(G)\}$ the width of spectrum of G. We have researched the case of $w_s(G) = 1$, namely finite groups with elements of prime power orders. This paper was published in Journal of Yunnan Education College, 1(1986) (in Chinese), also see arXiv: 2003.09445. In this talk, we review this article, discuss some special cases of $w_o(G)$ and $w_s(G)$, and pose some new problems.

Generalised Sakuma theorem

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In 2007 Sakuma published a theorem classifying a class of vertex operator algebras (VOA) generated by two Ising vectors. An Ising vector v is a conformal vector of weight 2; that is, v generates a Virasoro sub VOA of central charge $\frac{1}{2}$. This concept was introduced by Miyamoto who observed that, because the fusion algebra for such a sub VOA is Z_2-graded, this grading extends to the whole VOA and so each Ising vector v to an automorphism τ_v of order 2 called the Miyamoto involution.

In the most relevant example of the Moonshine VOA $V^\natural$, whose automorphism group is the sporadic finite simple group M, the Monster, and whose weight 2 component $V = V_2^\natural$ is the Griess algebra used to construct M, the Ising vectors are the same as Norton's $2A$-axes and the Miyamoto involutions are the same as the $2A$ involutions from M.

Sakuma Theorem states that VOAs generated by two Ising vectors are of eight types all represented as sub VOA of $V^\natural$ and indexed by the classes of M whose elements are products of two $2A$ involutions: $2A$, $2B$, $3A$, $3C$, $4A$, $4B$, $5A$, and $6A$. In 2010 Ivanov, Pasechnik, Seress, and Shpectorov generalised Sakuma's result to the class of Majorana algebras introduced by Ivanov in 2009. This was further generalised in 2015 by Hall, Rehren, and Shpectorov to a wider class of axial algebras with the fusion rules as in the Griess algebra and additionally admitting a form associating with the algebra product. In the same year Rehren attempted the ultimate generalisation dropping the form condition and allowing arbitrary α and β instead of the specific values $\frac{1}{4}$ and $\frac{1}{32}$ arising in the fusion rules for the Griess algebra. He obtained an upper bound of eight on the dimension of the 2-generated algebra, provided that $\alpha \neq 2\beta$ or 4β.

In the lecture we will discuss the recent results, joint with Franchi and Mainardis, in the general program of classifying 2-generated algebras of Monster type (α, β). This includes the result for $(\alpha, \beta) = (\frac{1}{4}, \frac{1}{32})$, the classification of the generic case, where α and β are (algebraically independent) indeterminates, the generic case with $\alpha = 2\beta$, β an indeterminate, and the infinite dimensional example arising for $(\alpha, \beta) = (2, \frac{1}{2})$. Note also that very recently Yabe completed the symmetric case; that is, where the algebra admits an automorphism switching the two generating axes.

Words separation and positive identities in symmetric groups

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We study short positive (i.e., containing no inverses) identities of finite symmetric groups. The interest in such identities is inspired by the problem of separating words with finite automata, in particular, with the automata in which each letter acts on the set of states as a permutation.

The main problem is to estimate the asymptotic length of the shortest positive identity in the symmetric group S_n. We present both theoretic and computer-assisted results towards the solution of this problem.

On axial algebras of Jordan type

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This is joint work with Ilya Gorshkov

The concept of axial algebras was introduced by Hall, Rehren, and Shpectorov [1]. These algebras are commutative, nonassociative, and generated by idempotents. Axial algebras extend the class of Majorana algebras, which were introduced as a part of Majorana theory by Ivanov [2]. The key motivational example for both theories is the Griess algebra, which is a real commutative nonassociative algebra of dimension 196884 that has the Monster sporadic simple group as its automorphism group. It is known that this algebra is generated by idempotents that correspond to involutions from the conjugacy class 2A of the Monster.

Axial algebras of Jordan type η are generated by idempotents whose adjoint operators have the minimal polynomial dividing $(x-1)x(x-\eta)$, where $\eta \notin \{0,1\}$ is fixed, with restrictive multiplication rules. These properties generalize the Peirce decompositions for idempotents in Jordan algebras, where $\frac{1}{2}$ is replaced with η. In particular, Jordan algebras generated by idempotents are axial algebras of Jordan type $\frac{1}{2}$. If $\eta \neq \frac{1}{2}$ then it is known that axial algebras of Jordan type η are factors of the so-called Matsuo algebras corresponding to 3-transposition groups. We call the generating idempotents *axes* and say that an axis is *primitive* if its adjoint operator has 1-dimensional 1-eigenspace. It is known that a subalgebra generated by two primitive axes has dimension at most three. The 3-generated case has been opened so far. In this talk, we discuss recent results on axial algebras of Jordan type generated by three primitive axes [3].

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Cayley graphs among vertex-symmetric graphs

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In the talk we consider Cayley graphs of groups as elements of the metric space $\mathfrak{G}$ of connected locally finite vertex-symmetric graphs (considered up to isomorphism; the distance between two non-isomorphic graphs in $\mathfrak{G}$ is 2^{-r} where r is the maximal radius of isomorphic balls of these graphs). In particular, using a rather general method of constructing Cayley graphs with trivial vertex stabilizers, we give an example of an infinite Cayley graph which is isolated in the space $\mathfrak{G}$. We also give examples of Cayley graphs which are not isolated in this space but are isolated from the set of connected vertex-symmetric finite graphs.

Computational complexity of synchronization under regular constraints

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Many variations of synchronization of finite automata have been studied in the previous decades. Here, we suggest studying the question if synchronizing words exist that belong to some fixed constraint language, given by some partial finite automaton called constraint automaton. We show that this synchronization problem becomes PSPACE-complete even for some constraint automata with two states and a ternary alphabet.

Digraph homomorphisms with the path-lifting property

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This is joint work with Yinfeng Zhu

For a digraph P, we say that a digraph homomorphism ϕ from G to H has the P-lifting property if for any homomorphism τ from P to H there exists a digraph homomorphism γ such that $\tau = \phi \circ \gamma$. A digraph homomorphism is called path-liftable if it has the P-lifting property for all path digraphs P.

If H is a strongly connected digraph but not a cycle, we show that it is NP-complete to decide whether or not a homomorphism from a digraph to H is path-liftable. For two strongly connected digraphs G and H of the same spectral radius, we give a cubic time algorithm to test whether or not a homomorphism from G to H is path-liftable. We show that a digraph homomorphism ϕ from an n-vertex digraph G to another digraph is path-liftable if it has the P-lifting property for all path digraphs P of length at most $2^n - 1$. If G and H are two strongly connected digraphs such that $|V(G)| = p|V(H)|$ for some prime number p, we provide a characterization of all path-liftable digraph homomorphisms from G to H.

Two projects in the representation theory of finite simple groups

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We discuss some regularities concerning the minimum polynomial degree and eigenvalue 1 of elements of irreducible finite linear groups, mainly quasi-simple.

The minimum polynomial degree project aims to obtain a best possible extension of the Hall-Higman theorem for p-solvable groups.

The eigenvalue project aims to determine the irreducible representations of finite simple groups in which every group element has eigenvalue 1.

We shall give a short overview of these projects and state some recent results.

Contributed Talks

On flag-transitive automorphism groups of 2-designs with $\gcd(r, \lambda) = 1$

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The main aim of this talk is to present a classification of 2-designs with $\gcd(r, \lambda) = 1$. In 1988, Zieschang [11] proved that if an automorphism group of a 2-design with $\gcd(r, \lambda) = 1$ is flag-transitive, then it is point-primitive of almost simple or affine type. Such designs admitting an almost simple automorphism group have been studied in [1–3, 7–10]. The case where a 2-design with $\gcd(r, \lambda) = 1$ admits an affine type automorphism group has been studied in [4–6]. In conclusion, all 2-designs with $\gcd(r, \lambda) = 1$ admitting flag-transitive automorphism groups are known except for those admitting one dimensional affine type automorphism groups.

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On the extremal problems concerning some bond incident degree indices of graphs

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The bond incident degree (BID) indices of graphs are the graph invariants satisfying some constraints. Many graph invariants that have found applications in chemistry are the BID indices. Finding graphs from the specific classes of graphs with extremum values of certain graph invariants is one of the much studied problems in chemical graph theory. In this talk, extremal results concerning some of the well-known BID indices, namely the general zeroth-order Randić index, general sum-connectivity index, symmetric division deg index and the augmented Zagreb index, will be dispensed.

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Donovan's conjecture and the classification of blocks

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Donovan's conjecture predicts that given a p-group D there are only finitely many Morita equivalence classes of blocks of group algebras with defect group D. While the conjecture is still open for a generic p-group D, it has been proven in 2014 by Eaton, Kessar, Külshammer and Sambale when D is an elementary abelian 2-group, and in 2018 by Eaton and Livesey when D is any abelian 2-group. The proof, however, does not describe these equivalence classes explicitly.

A classification up to Morita equivalence over a complete discrete valuation ring $\mathcal{O}$ has been achieved for D with rank 3 or less, and for $D = (C_2)^4$.

I have done $(C_2)^5$, and I have partial results on $(C_2)^6$. In this talk I will introduce the topic, give the relevant definitions and then describe the process of classifying this blocks, with a particular focus on the methodology and the individual tools needed to achieve a complete classification.

On the 486-vertex distance-regular graphs of Koolen–Riebeek and Soicher

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This is joint work with Daniel Hawtin (Rijeka)

In this talk, we consider three imprimitive distance-regular graphs with 486 vertices and diameter 4: the Koolen–Riebeek graph (which is bipartite), the Soicher graph (which is antipodal), and the incidence graph of a symmetric transversal design obtained from the affine geometry $\mathrm{AG}(5,3)$ (which is both). We will show that each of these is preserved by the same rank-9 action of the group $3^5 : (2 \times M_{10})$, and the connection is explained using the ternary Golay code.

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Group Actions on Invariant Relations

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Group invariant relations have been an important tool in the study of permutations groups, see for instance Wielandt's Ohio State lecture notes [9]. Group invariant relations are equally important for the constructive theory of combinatorial objects. In this talk, we will discuss examples of group invariant relations and also point out ways to make use of these relations for designing algorithms to classify combinatorial objects efficiently. By stacking many relations together, we can classify the orbits of a group on a poset, which we call poset classification. This technique has been used by McKay [7] under the name canonical augmentation. Partition backtrack (see Leon [6]) is yet another way to classify objects, though it can be costly. The need for backtracking and canonical forms has been eliminated by work of Schmalz [8], using a trade between time and memory. This has been used and refined by the author, see [2]. Some applications of this new technique to the classification of cubic surfaces with 27 lines over finite fields will be given [3]. Performance comparisons between some major systems (GAP [5], Magma [4], Orbiter [1]) will be discussed as well.

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Finite groups with an affine map of large order

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Let G be a group. An *affine map of* G is a function $G \to G$ of the form $g \mapsto g^{\alpha}t$ for a fixed automorphism α and a fixed element t of G. The affine maps of G form a permutation group on G known as the *holomorph of* G, and they arise naturally both in theoretical and applied contexts. For example, several of the infinite classes of finite primitive permutation groups listed in the celebrated O'Nan-Scott theorem can be viewed as subgroups of holomorphs (more precisely, these are the classes HA, HS and HC described in [4, Section 3]), and affine maps on abelian groups have been used as a simple yet effective means of creating pseudorandom number sequences for a long time (for example in linear congruential generators, described in [3, pp. 168ff.], or in vector generators, described in [3, pp. 205ff.]).

In this talk, we are concerned with finite groups G admitting an affine map of "large"order, say at least $\rho|G|$ for a fixed $\rho \in (0,1]$ – we note that an earlier result of the author, [1, Theorem 5.2.3], implies that the order of an affine map of G cannot exceed $|G|$. This condition is loosely motivated by pseudorandom number generation, where the permutations used to create pseudorandom sequences need to have at least one "large"cycle (to avoid early repetitions in the sequence). Another earlier result of the author, [1, Theorem 1.1.3(2)], states that in a finite group G with an affine map of order at least $\rho|G|$, the largest solvable normal subgroup of G, the solvable radical $\mathrm{Rad}(G)$, has index at most $\rho^{-5.91}$ in G. Here, we discuss recent results from the preprint [2], including the statement that the derived length of $\mathrm{Rad}(G)$ is at most $4 \cdot \lfloor \log_2(\rho^{-1}) \rfloor + 3$.

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Combinatorially defined point sets in finite projective planes

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A $(q+t)$-set $\mathcal{K}$ of type $(0,2,t)$ is a point set of size $q+t$ in a finite projective plane of order q meeting each line in 0, 2 or in t points. Note that if $t \neq 2$ then this means that through each point of $\mathcal{K}$ there passes a unique line meeting $\mathcal{K}$ in t points. For $t=1$ we get the *ovals*, for $t=2$ the *hyperovals*; thus this concept generalizes well-known objects of finite geometry. Such point sets were studied first by Korchmáros and Mazzocca in 1990, see [3], that is why nowadays they are called *KM-arcs*. KM-arcs exist only for q and t even and they have been studied mostly in $\mathrm{PG}(2,q)$, the Desarguesian projective plane of order q, where Gács and Weiner proved that the t-secants (that is, lines meeting the KM-arc in t points) are concurrent, see [2]. Consider the following generalization:

A *generalized KM-arc of type* $(0,m,t)$ is a point set $\mathcal{S}$ of a finite projective plane of order q such that each point Q of $\mathcal{S}$ is incident with a line meeting $\mathcal{S}$ in t points and the other lines incident with Q meet $\mathcal{S}$ in m points. In [1] we proved the following characterisation of these objects:

Theorem 1. For a generalized KM-arc $\mathcal{S}$ of type $(0,m,t)$ in $\mathrm{PG}(2,q)$, $q=p^n$, p prime, either $m \equiv t \equiv 0 \pmod{p}$, or $\mathcal{S}$ is one of the following:

(1) a set of t collinear points ($m=1$),

(2) the union of m lines incident with a point P, minus P $(t=q)$,

(3) an oval ($t=1$, $m=2$),

(4) a maximal arc with at most one of its points removed ($t=m$, or $t=m-1$),

(5) a unital ($t=1$, $m=\sqrt{q}+1$),

(6) complement of a Baer subplane ($t=q-\sqrt{q}$, $m=q$).

The proof relies on a stability result of Szőnyi and Weiner regarding k mod p multisets, see [4], and on other polynomial techniques which ensure that in case of $t \not\equiv m \pmod{p}$ the t-secants meeting a fixed m-secant in $\mathcal{S}$ are concurrent.

I will also discuss some mod p generalizations and what happens if we weaken the conditions.

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Computation of various degree-based topological indices of the third type of triangular Hex-derived network of dimension n by using M-polynomial

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This is a joint work with Shikha Rai

A topological index is a numerical molecular descriptor that reports several physical properties, biological activities and chemical reactivities of a molecular graph to understand its behaviour. In recent studies, various degree-based topological indices of a molecular structure are calculated by deriving a M-polynomial of that structure. In this talk, we derive the M-polynomial of the third type of triangular Hex-derived network of dimension n and then estimate the corresponding degree-based topological indices. In addition, we pictorially represent the M-polynomial and the related degree-based topological indices for different dimensions.

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Gregarious Kite Factorization of Tensor Product of Complete Graphs

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This is joint work with A. Muthusamy

Partition of G into subgraphs G_1, G_2, $\ldots$, G_r such that $E(G_i) \cap E(G_j) = \emptyset$ for $i \neq j \in \{1, 2, \ldots, r\}$ and $E(G) = \cup_{i=1}^{r} E(G_i)$ is called *decomposition* of G; In this case we write G as $G = G_1 \oplus G_2 \oplus \ldots \oplus G_r$, where $\oplus$ denotes edge-disjoint sum of subgraphs. A spanning subgraph of G such that each component of it is isomorphic to some graph H is called an H*-factor* of G. A Partition of G into edge-disjoint H-factors is called an H-factorization of G.
A *kite* is a graph which is obtained by attaching an edge to a vertex of the triangle. We denote the kite with edge set $\{ab, bc, ca, cd\}$ by $(a, b, c; cd)$. A subgraph of a multipartite graph G is said to be *gregarious* if each of its vertices lies in different partite sets of G. A kite factorization of a multipartite graph is said to be gregarious if every kite in the factorization has all its vertices in different partite sets.

We show that there exists a gregarious kite factorization of $K_m \times K_n$ if and only if $mn \equiv 0 \pmod 4$ and $(m-1)(n-1) \equiv 0 \pmod 2$, where $\times$ denotes the tensor product of graphs.

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Graphs of diameter 2 and their diametral vertices

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In order to find the asymptotically exact value of the number of n-vertex graphs of fixed diameter properties of typical graphs of the class $\mathcal{J}_{n,\,d=k}$ of all n-vertex labeled graphs of diameter k are investigated in [1–3]. It was proved that almost all n-vertex graphs of a given diameter $k \geq 3$ have a unique pair of diametral vertices but for n-vertex graphs of diameter 2 such property is not valid. This prompted a detailed investigation of more general properties of graphs of diameter 2 related to the number of pairs of diametral vertices contained in the graph. In addition, the class $\mathcal{J}_{n,\,d=2}$ has always been of particular interest in view of, on the one hand, the apparent simplicity of its objects, on the other hand, the breadth of its “coverage” of all graphs: it is well known that almost all graphs have a diameter 2. In this connection, a more subtle classification of graphs of diameter 2 is interesting, when subclasses are distinguished that form a partition of the entire class $\mathcal{J}_{n,\,d=2}$. Moreover, for meaningful classification it is required that none of the considered subclasses would not be poor and too rich, i.e. asymptotically did not coincide with the whole class. At the same time, considering the "wealth" of the whole class $\mathcal{J}_{n,\,d=2}$, apparently we should not expect a good characterization of the selected subclasses, however, there is a natural problem of describing or constructing a class of typical graphs inside each studied subclass in order to clarify the structure of such graphs with a large number of vertices.

In the present talk the classification of graphs of diameter 2 by the number of pairs of diametral vertices contained in the graph is designed. All possible values of the parameters n and k are established for which there exists a n-vertex graph of diameter 2 that has exactly k pairs of diametral vertices. As a corollary, the smallest order of these graphs is found. Such graphs with a large number of vertices are also described and counted. In addition, for any fixed integer $k \geq 1$ inside each distinguished class $\mathcal{J}_{n,\,q=k}$ of n-vertex graphs of diameter 2 containing exactly k pairs of diametral vertices, a class of typical graphs is constructed. For the introduced classes, the almost all property is studied for any $k = k(n)$ with the growth restriction under consideration, covering the case of a fixed integer $k \geq 1$. As a consequence, it is proved that it is impossible to limit the number of pairs of diametral vertices by a given fixed integer k in order to obtain almost all graphs of diameter 2.

Теорема 1. *Let $k \geq 1$. Then there exists a graph containing exactly k pairs of diametral vertices in the class of n-vertex graphs of diameter 2 iff $n \geq \lceil 0.5(3 + \sqrt{1+8k}) \rceil$.*

Теорема 2. *Let $k \geq 1$ be any given integer. Then $|\mathcal{J}_{n,\,q=k}| = \left(\binom{\binom{n}{2}}{k}\right)$ for any $n > k+1$.*

Теорема 3. *Let $k \geq 1$ be any fixed integer. Then graphs isomorphic $\overline{kK}_2 + K_{n-2k}$ for $n \neq 2$ form a class of typical graphs of the class $\mathcal{J}_{n,\,q=k}$.*

Теорема 4. *Let $0 < k \leq \epsilon\binom{n}{2}$ for $n \to \infty$ where ϵ is fixed constant and $0 < \epsilon < \frac{1}{2}$. Then almost all n-vertex graphs of diameter 2 have at least k pairs diametral vertices.*

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On the set related to the rank of a Sylow p-subgroup in finite groups

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In this paper, we use the properties of Burnside basis of Sylow subgroups to study the structure of finite groups. In particular, we get some results for p-supersolvable groups and p-Fitting subgroup.

Groups generated by derangements

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Given a permutation group G we can study the subgroup $D(G)$ generated by all the derangements in G. In this talk I will discuss recent joint work with Rosemary Bailey, Peter Cameron and Gordon Royle which studies $D(G)$ and in particular bounds the index $|G : D(G)|$ and looks at the structure of the quotient group $G/D(G)$.

Characterization of some finite G-graphs

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G-graph and Cayley graph have both nice and highly regular properties, but the G-graphs are not always vertex-transitive. We try to characterize some finite G-graphs.

Unipotent dynamics on a torus

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This is joint work with Ivan Mitrofanov and Prof. Alexey Kanel-Belov

In this paper we investigate unipotent dymamics on a torus and apply it to the following problem. For an integer k, consider a sequence of digits (a_n), where a_n is the first digit in the decimal representation of 2^{n^k}. How to find the number of subwords of (a_n) of a given length?

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On distance-regular graphs with intersection array {7(n-1),6(n-2),4(n-4);1,6,28}

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This is joint work with Alexander A. Makhnev and Ivan N. Belousov

There is an infinite sequence of formally self-dual classical distance-regular graphs Γ with classical parameters $b = 2$, $\alpha = 1$, $\beta = n-1$, $v = n^3$ $(n > 5)$ (see, for example, [1]). If n is a power of 2, then there exists a distance-regular graph Γ with intersection array $\{7(n-1), 6(n-2), 4(n-4); 1, 6, 28\}$ and each distance-regular graph with these parameters is a bilinear forms graph.

K. Metsch [2] has proved that if a distance-regular graph with intersection array $\{7(n-1), 6(n-2), 4(n-4); 1, 6, 28\}$ is not a bilinear forms graph, then $n \leq 133$. In this work, we improve the corresponding estimation to $n \leq 70$ for an arbitrary distance-regular graph with intersection array $\{7(n-1), 6(n-2), 4(n-4); 1, 6, 28\}$, and to $n \leq 42$, for geometric graphs.

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Equitable partitions of a generalized Petersen graph into 3 cells

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A joint work with Danil V. Kerkesner, *Novosibirsk State University*, d.kerkesner@g.nsu.ru

We consider the generalized Petersen graph $G(n, 2)$ and study its equitable partitions into 3 cells. All partition matrices are enumerated for this case. For each matrix, we supply equitable partitions implementing this matrix and obtain conditions necessary and sufficient for existing of such partitions.

The generalized Petersen graph $G(n, k)$, $k \le n/2$, is a graph with the vertex set $V = \{u_0, u_1, \dots, u_{n-1}, v_0, v_1, \dots, v_{n-1}\}$ and edge set $E = \{u_i u_{i+1}, u_i v_i, v_i v_{i+k} \mid 0 \le i \le n-1\}$ where subscripts to be read modulo n.

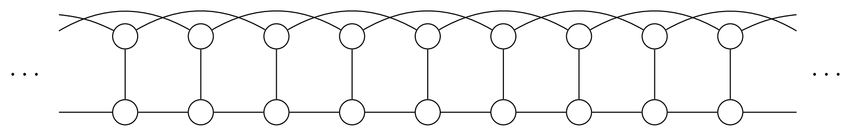

Figure 1: The generalized Petersen graph $G(n, 2)$

Avgustinovich and Lisitsyna [1] described equitable partitions into 2 cells of some transitive cubic graphs. Among other results they enumerated all partition matrices that the generalized Petersen graph $G(n, 2)$ admits for its equitable partitions into 2 cells:

$$A_1 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \quad (n \in \mathbb{N}), \quad A_2 = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} \quad (3 \mid n), \quad A_3 = \begin{pmatrix} 0 & 3 \\ 2 & 1 \end{pmatrix} \quad (5 \mid n).$$

In this talk we continue to study $G(n, 2)$ and discuss equitable partitions of its vertex set into 3 cells.

A partition $P = \{C_0, C_1, \dots, C_{\ell-1}\}$ of the vertex set V is *equitable* if for every pair of (not necessarily distinct) indices $i, j \in \{0, 1, \dots, \ell-1\}$ there is an integer $a_{ij} \ge 0$ such that each vertex u in the *cell* C_i has exactly a_{ij} neighbors in C_j. The matrix $A = (a_{ij})$ is called the *partition matrix*.

It is easy to see that the vertex transform $\varphi\colon u_i \mapsto u_{i+1}$, $v_i \mapsto v_{i+1}$ is an automorphism of the graph $G(n, k)$. In this way, we call the partition P to be *periodic* if there is a *period* $T \in \mathbb{N}$ such that $\varphi^T(P) = P$, i. e., $\varphi^T(C_i) = C_i$ for each $i \in \{0, 1, \dots, \ell-1\}$.

Theorem 1. An arbitrary equitable partition of the generalized Petersen graph $G(n, 2)$ into 3 cells has one of the following partition matrices:

$$M_1 = \begin{pmatrix} 2 & 1 & 0 \\ 2 & 0 & 1 \\ 0 & 3 & 0 \end{pmatrix}, \quad M_2 = \begin{pmatrix} 1 & 2 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 1 \end{pmatrix}, \quad M_3 = \begin{pmatrix} 0 & 3 & 0 \\ 1 & 1 & 1 \\ 0 & 3 & 0 \end{pmatrix}, \quad M_4 = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 0 \end{pmatrix}, \quad M_5 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

Theorem 2. For the generalized Petersen graph $G = G(n, 2)$ the following statements hold.
(1) G admits an equitable partition with the matrix $M \in \{M_1, M_2\}$ iff n is a multiple of 5.
(2) G admits an equitable partition with the matrix M_3 iff n is a multiple of 10.
(3) G admits an equitable partition with the matrix $M \in \{M_4, M_5\}$ iff n is a multiple of 3.

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On coincidence of Gruenberg–Kegel graphs of non-isomorphic finite groups

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We use mainly standard notation and terminology (see [1]). Let G be a finite group. Denote by $\pi(G)$ the set of all prime divisors of the order of G and by $\omega(G)$ the *spectrum* of G, i.e., the set of all its element orders. The set $\omega(G)$ defines the *Gruenberg-Kegel graph* (or the *prime graph*) $\Gamma(G)$ of G; in this simple graph the vertex set is $\pi(G)$, and distinct vertices p and q are adjacent if and only if $pq \in \omega(G)$.

The problem of description of the cases when Gruenberg-Kegel graphs of non-isomorphic finite groups coincide naturally arises. One of the most interesting cases of this problem is when Gruenberg–Kegel graphs of non-isomorphic finite groups are both disconnected and coincide.

Recall some known definitions. A finite group G is a *Frobenius group* if there is a non-trivial proper subgroup C of G such that $C \cap gCg^{-1} = \{1\}$ whenever $g \notin C$. A subgroup C is called a *Frobenius complement* of G. Let $K = \{1\} \cup (G \setminus \bigcup_{g\in G} gCg^{-1})$. Then K is a normal subgroup of a Frobenius group G with a Frobenius complement C which is called the *Frobenius kernel* of G. A finite group G is a *2-Frobenius group* if $G = ABC$, where A and AB are normal subgroups of G, AB and BC are Frobenius groups with kernels A and B and complements B and C, respectively. It is known that each 2-Frobenius group is solvable. The *socle* $Soc(G)$ of a finite group G is the subgroup of G generated by all its non-trivial minimal normal subgroups. A finite group G is *almost simple* if $Soc(G)$ is a finite non-abelian simple group.

By the *Gruenberg–Kegel Theorem*, if G is a finite group with disconnected Gruenberg-Kegel graph, then either G is a Frobenius group or either G is a 2-Frobenius group or G is an extension of a nilpotent group by an almost simple group. In [2], all the cases of coincidence of Gruenberg-Kegel graphs of a finite simple group and of a Frobenius or a 2-Frobenius group were described. Moreover, we can obtain the complete list of almost simple groups whose Gruenberg-Kegel graphs coincide with Gruenberg-Kegel graphs of solvable Frobenius groups or 2-Frobenius groups directly from the main results of the papers [3] and [4].

In this talk, we concentrate on obtaining a list of almost simple (but not simple) finite groups whose Gruenberg-Kegel graphs coincide with Gruenberg-Kegel graphs of non-solvable Frobenius groups. We prove the following

Theorem. *Let G be an almost simple but not simple finite group. If $\Gamma(G) = \Gamma(H)$, where H is a non-solvable Frobenius group, then one of the following statements holds:*

(1) *G is isomorphic to one of the groups $Aut(M_{12})$, S_7, ${}^2F_4(2)$, $Aut(PSU_4(2))$, $L_3(4).2_3$, $PSL_4(3).2_2$, $PSL_4(3).2_3$, $PSL_2(11).2 \cong PGL_2(11)$, $PSL_2(19).2 \cong PGL_2(19)$, $PSL_2(49).2_1 \cong PGL_2(49)$, $PSL_2(25).2_2$, $PSL_2(81).2_1$, $PSL_2(81).4_1$, or $PSL_2(81).4_2$;*

(2) $Soc(G) \in \{PSL_3(q), PSU_3(q)\}$.

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Subgroups of arbitrary even ordinary depth

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The notion of depth was originally defined for von-Neumann algebras, see [2]. Later it was also defined for Hopf algebras, see [8]. For some recent results in this direction, see [3–5]. In [7] and later in [1], the depth of semisimple algebra inclusions was studied, by Burciu, Kadison and Külshammer. First results were considering the depth 2 case, later it was generalized for arbitrary n. In the case of group algebra inclusion $\mathbf{C}H \subseteq \mathbf{C}G$ it was shown that the depth is at most 2 if and only if H is normal in G, see [7]. It is shown in [1] that the depth of the symmetric group S_n in S_{n+1} is $2n-1$. We show that for each positive integer n, there exist a group G and a subgroup H such that the ordinary depth is $d(H,G) = 2n$. This solves the open problem posed by Lars Kadison (see [6]) whether even ordinary depth larger than 6 can occur.

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A short explicit proof of Greenberg's Theorem

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In 1960 Greenberg [2] proved that every countable group A is isomorphic to the automorphism group of a Riemann surface $\mathcal{S}$, and in 1973 he proved that if A is finite then $\mathcal{S}$ can be chosen to be compact [3]. The proofs are long, complicated and not constructive. In response to a question by Sasha Mednykh, I will give a short and explicit algebraic proof of Greenberg's Theorems for finitely generated groups A, using results of Macbeath [5], Singerman [7] and Takeuchi [8] on triangle groups, and of Margulis [6] on arithmeticity. In each case, $\mathcal{S} = \mathbb{H}/M$ where $\mathbb{H}$ is the hyperbolic plane and M ($\cong \pi_1\mathcal{S}$) is an explicit subgroup of a triangle group acting on $\mathbb{H}$. When A is finite it follows from this and from Belyĭ's Theorem [1] that $\mathcal{S}$ can be defined, as a complex algebraic curve, over an algebraic number field. For full details, see [4].

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On chief factors of parabolic maximal subgroups of the group $B_l(2^n)$

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One of the fundamental problems of group theory is the study of the subgroup structure of a given group. In the post-classification period, investigating subgroups and representations of finite simple groups has become an actual problem. The important class of permutation representations of finite groups of Lie type is given by its parabolic representations, i.e., the representations on the cosets of parabolic subgroups. In our previous works, we obtained a description for the primitive parabolic permutation representations of all groups of Lie type (normal and twisted).

The author continues to study the properties of these representations, namely, the chief factors of parabolic maximal subgroups in finite simple groups of Lie type are studied.

Let G be a finite simple group of Lie type over a finite field K of characteristic p and let $P = UL$ be a parabolic maximal subgroup in G, where U is the unipotent radical and L is the Levi complement in P. Suppose that $p \neq 2$ for the groups G of type B_l, C_l, or F_4 and $p > 3$ for the groups G of type G_2. Then the results of [1] imply that the factors of the lower central series of U are chief factors of P and irreducible KL-modules. The number of these factors is independent of the field K but depends on the Lie type of G. In the exceptional cases, the commutator relations influencing the structure of unipotent subgroups behave in a special way and require special consideration.

In the previous papers [2]– [7], the author obtained a refined description of chief factors of parabolic maximal subgroups involving in the unipotent radical for all groups of Lie type, except for the groups $B_l(2^n) \cong C_l(2^n)$.

In the present work, such description for $B_l(2^n)$ is given. It is proved a theorem, in which, for every parabolic maximal subgroup of the group $B_l(2^n)$, a fragment of its chief series that involves in the unipotent radical of this parabolic subgroup is given. Generating elements of the corresponding chief factors are presented in a table.

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Local-global invariants of groups and Lie algebras

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Given a group G, one can look at its conjugation action on itself, consider the first (nonabelian) cohomology set $H^1(G, G)$, and define the Shafarevich–Tate set $Ш(G)$ as the subset of $H^1(G, G)$ consisting of the cohomology classes becoming trivial after restriction to every cyclic subgroup $C \subset G$ [6].

If G is finite, $Ш(G)$ is a finite group. It coincides with the group $\mathrm{Out_c}(G) = \mathrm{Aut_c}(G)/\mathrm{Inn}(G)$ of outer class-preserving automorphisms of G, introduced more than 100 years ago by Burnside [2].

For a finite group G, one can consider the Schur multiplier $H^2(G, \mathbb{Q}/\mathbb{Z})$ and define the Bogomolov multiplier $B(G)$ as the subset of $H^2(G, \mathbb{Q}/\mathbb{Z})$ consisting of the cohomology classes becoming trivial after restriction to every bicyclic subgroup $A \subset G$ [1]. One can extend this definition to the case of infinite groups [1], [5].

Various properties of these invariants are discussed in [3], [4]. We will give a brief survey of some recent developments, including interrelations between the two invariants and their Lie-algebraic analogues.

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On small antipodal graphs of diameter 4

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Let Γ be a distance-regular graph of diameter 4. If $\Gamma_{3,4}$ is a strongly regular graph then to determine the intersection array of Γ by parameters of $\Gamma_{3,4}$ is an inverse problem.

Known examples of primitive graphs are the following (see [1]).

1. Odd graph $\Gamma = O_9$ has intersection array $\{5, 4, 4, 3; 1, 1, 2, 2\}$ and $\Gamma_{3,4}$ has parameters (126,100,78,84).

2. Folded 9-cube Γ has intersection array $\{9, 8, 7, 6; 1, 2, 3, 4\}$ and $\Gamma_{3,4}$ has parameters (256,210,170,182).

3. Dual polar graph Γ has intersection array $\{30, 28, 24, 16; 1, 3, 7, 15\}$ and $\Gamma_{3,4}$ has parameters (2295,1984,1708,1860).

The exists the unique bipartite antipodal distance-regular graph Γ with strongly regular graph $\Gamma_{3,4}$ (see p. 425 in [1]):

4. 4-cube Γ has intersection array $\{4, 3, 2, 1; 1, 2, 3, 4\}$ and $\Gamma_{3,4}$ has parameters (16,5,0,2).

Theorem 1 [2]. Let Γ be an antipodal distance-regular graph of diameter 4 with strongly regular graph $\Delta = \Gamma_{3,4}$. Then $\lambda(\Delta) = 0$, $b_0 = k(\Delta) - 1$, $c_2 = a_1 + 2 = \mu(\Delta)$ and $b_1 = k(\Delta) - \mu(\Delta)$.

In this talk, we discuss small antipodal distance-regular graphs Γ of diameter 4.

Theorem 2. Let a distance-regular graph with intersection array $\{56, 45, 24(r-1)/r, 1; 1, 24/r, 27, 32\}$, $r \in \{2, 3, 4, 6, 8\}$ exists. Then $r = 3$.

Theorem 3. If a distance-regular graph Γ with intersection array $\{96, 75, 32(r-1)/r, 1; 1, 32/r, 75, 96\}$ exists, then $r = 2$, Γ is not a locally $GQ(5, 3)$-graph, and the group $G = \mathrm{Aut}(\Gamma)$ acts intrasitively on the set of antipodal classes of Γ.

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On the lattice of ω-fibered formations of finite groups

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This is joint work with Marina Sorokina

Only finite groups are considered. In the theory of the classes of finite groups, much attention has been paid to the research of the lattice properties of the classes. Nowadays many important properties of the lattices of local and ω-local formations and Fitting classes have been established (see, for instance, [1], [2]). An ω-local formation is one of the types of ω-fibered formations (see [3]). In the theorems $1-3$, we have obtained some properties of the lattice of ω-fibered formations with the definite direction δ.

We use standard definitions and notations for groups, classes of groups, and lattices (see, for instance, [1], [4]). Let π be a non-empty subset of the set $\mathbb{P}$ of all primes. Recall that a group G is π-solvable if $|\pi(H/K)| = 1$ for every chief πd-factor H/K of G; π-selected if $|\pi(H/K) \cap \pi| \leq 1$ for every chief factor H/K of G [5]. The classes of all identity groups, all p-groups and all p'-groups for $p \in \mathbb{P}$ are denoted by (1), $\mathfrak{N}_p$ and $\mathfrak{E}_{p'}$, respectively. Let ω be a non-empty subset of the set $\mathbb{P}$, $f : \omega \cup \{\omega'\} \to \{$formations of groups$\}$ be an ωF-function, $\delta : \mathbb{P} \to \{$non-empty Fitting formations$\}$ be a $\mathbb{P}FR$-function. A formation $\omega F(f, \delta) = (G : G/O_\omega(G) \in f(\omega')$ with $G/G_{\delta(p)} \in f(p)$ for all $p \in \omega \cap \pi(G))$ ($O_\omega(G)$ and $G_{\delta(p)}$ are the largest normal ω-subgroup of the group G and the $\delta(p)$-radical of G, respectively) is called an ω-fibered formation with the d irection δ (see [3]. By δ_0, we denote the $\mathbb{P}FR$-function such that $\delta_0(p) = \mathfrak{E}_{p'}$ for every $p \in \mathbb{P}$. A $\mathbb{P}FR$-function δ is called b-direction if $\delta(p)\mathfrak{N}_p = \delta(p)$ for every $p \in \mathbb{P}$; δ is called s-direction if $\delta(p)$ is a p-solvable class for every $p \in \mathbb{P}$ (see [6]). Denote by $\Theta_{\omega\delta}$ the set of all ω-fibered formations with the direction δ, by $\Theta_{\omega\delta}(\mathfrak{F})$ the set of all ω-fibered subformations with the direction δ of the formation $\mathfrak{F}$. It has been well known that, in the case $\delta_0 \leq \delta$, the sets $\Theta_{\omega\delta}$ and $\Theta_{\omega\delta}(\mathfrak{F})$ are the lattices.

Theorem 1. *Let δ be a b-direction, $\delta_0 \leq \delta$, and suppose that $(1) \neq \mathfrak{F} \in \Theta_{\omega\delta}$. Then $|\Theta_{\omega\delta}(\mathfrak{F})| = 2$ if and only if $\mathfrak{F}$ is an ω-fibered formation generated by the simple ω-solvable group.*

Theorem 2. *Let δ be a b-direction, $\delta_0 \leq \delta$, and let $\mathfrak{F}$ be an ω-fibered formation with the direction δ generated by the simple ω-selected group. Then $|\Theta_{\omega\delta}(\mathfrak{F})| \leq 3$.*

Theorem 3. *Let $\mathfrak{F}$ be an one-generated ω-fibered formation with the direction δ, $\delta_0 \leq \delta$. The lattice $\Theta_{\omega\delta}(\mathfrak{F})$ has finite number of atoms if at least one of the following conditions is satisfied*:
(1) *$\mathfrak{F}$ is an ω-solvable class*;
(2) *δ is an s-direction.*

These theorems imply the results for such types of ω-fibered formations as ω-local, ω-special, ω-central formations.

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Classification of maximal subgroups of odd index in finite almost simple groups and some its applications

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M. Liebeck and J. Saxl [4] and, independently, W. Kantor [2] proposed a classification of finite primitive permutation groups of odd degree. Both papers [4] and [2] contain lists of subgroups of finite almost simple groups that can turn out to be maximal subgroups of odd index. However, in the case when the socle of an almost simple group is an alternating group or a classical group over a field of odd characteristic, neither in [4] nor in [2] it was described which of the specified subgroups are precisely maximal subgroups of odd index. Thus, the problem of the complete classification of maximal subgroups of odd index in finite almost simple groups remained open.

For alternating and symmetric groups, we completed the classification in [6]. For simple classical groups over fields of odd characteristics, the classification was completed in [5]. For almost simple groups with classical socle of dimension at least 13, the classification was completed in [7, 8]. In [5] we used results obtained by P. Kleidman in [3]. However there is a number of mistakes and inaccuracies in [3]. These mistakes and inaccuracies were corrected in [1]. In [9], we revised of the classification of maximal subgroups of odd index in finite simple classical groups obtained in [5]. Recently, we have completed the classification of maximal subgroups of odd index in almost simple groups with classical socle of degree at most 12 over a field of odd characteristic[1]. Thus, the classification of maximal subgroups of odd index in almost simple groups with classical socle is complete. In this talk, we discuss these results as well as some applications of the complete classification of maximal subgroups of odd index in finite almost simple groups.

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On finite non-solvable groups whose Gruenberg–Kegel graphs are isomorphic to the paw

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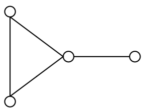
This is joint work with Anatoly Kondrat'ev

We use mainly standard notation and terminology (see [1]).

Let G be a finite group. Denote by $\pi(G)$ the set of all prime divisors of the order of G. The Gruenberg–Kegel graph (or the prime graph) $\Gamma(G)$ of G is a graph with the vertex set $\pi(G)$, in which two distinct vertices p and q are adjacent if and only if there exists an element of order pq in G.

A.S. Kondrat'ev described finite groups with the same Gruenberg-Kegel graph as the groups $Aut(J_2)$ [2] and A_{10} [3], respectively. The Gruenberg–Kegel graphs of all these groups are isomorphic as abstract graphs to the paw having the following form:



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We establish a more general problem: *to describe finite groups whose Gruenberg-Kegel graphs are isomorphic as abstract graphs to the paw.*

As a part of the solution of this problem, we have proved in [4] that if G is a finite non-solvable group and the graph $\Gamma(G)$ as abstract graph is isomorphic to paw, then the quotient group $G/S(G)$ (where $S(G)$ is the solvable radical of G) is almost simple, and classified all finite almost simple groups whose the Gruenberg-Kegel graphs as abstract graphs are isomorphic to subgraphs of the paw.

In the talk we discuss some new our results on this problem.

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Counting Schur Rings over Cyclic Groups

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Any Schur ring, an algebraic structure closely related to association schemes, is uniquely determined by a partition of the elements of the group. An open question in the study of Schur rings is determining which partitions of the group induce a Schur ring. Although a structure theorem is available for Schur rings over cyclic groups, it is still a difficult problem to count all the partitions. For example, Kovбcs, Liskovets, and Pцschel determine formulas to count the number of wreath-indecomposable Schur rings. In this talk we solve the problem of counting the number of all Schur rings over cyclic groups for various orders and draw some parallels with Higman's PORC conjecture.

Transitive extended perfect codes from regular subgroups of $GA(r,2)$

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The vector space of dimension n over $GF(2)$ is denoted by F^n. Consider the transformation (x,π), where $x \in F^n$ and π is a permutation of the positions of F^n that maps a binary vector y as $(x,\pi)(y) = x + \pi(y)$. The *automorphism group* $Aut(F^n)$ of F^n w.r.t. Hamming metric is defined as the group of all such transformations (x,π) with respect to the composition. Codes C and D are *isomorphic* if there are $x \in F^n$ and $\pi \in S_n$ such that $x + \pi(C) = D$. *The automorphism group* $Aut(C)$ of a code C is the setwise stabilizer of C in $Aut(F^n)$. A code C is *transitive* if $Aut(C)$ acts transitively on the set of its codewords.

Gillespie and Praeger [1] suggested the following concept. A code C is called *k-neighbor transitive* if for any i, $0 \le i \le k$, $Aut(C)$ acts transitively on C_i, where C_i is the set of words at distance exactly i from C. The extended Hamming code is 2-neighbor transitive and the Mollard construction [2] applied to 1-neighbor transitive extended perfect codes gives a 1-neighbor transitive extended perfect code.

The general affine group $GA(r,2)$ of the vector space F^r consists of mappings that could be represented by all pairs (a,M): $(a,M)(b) = a + Mb$ for a vector $a \in F^r$, $M \in GL(r,2)$. A subgroup G of $GA(r,2)$ is called *regular* if it is regular with respect to this action. By the definition for any regular subgroup of $GA(r,2)$ and any $a \in F^r$ there is a unique affine transformation from G that maps the all-zero vector $\mathbf{0}$ to a, which we denote by g_a. Let T be an automorphism of a regular subgroup of $GA(r,2)$. By τ we denote the permutation on the vectors of F^r *induced* by the automorphism T, i.e. $T(g_a) = g_{\tau(a)}$.

Main construction. Let the coordinates of F^{2^r} be indexed by the vectors of F^r. Define an extended Hamming code of length 2^r as follows: $\mathcal{H} = \{x \in F^{2^r} : \sum_{a\in F^r : x_a = 1} a = \mathbf{0}, \sum_{i\in\{1,\dots,n\}} x_i = 0\}$. Let $u|v$ denote the concatenation of vectors u and v. Consider a version of the Solov'eva construction [4]:

$$S_\tau = \bigcup_{a\in F^r, h, h' \in \mathcal{H}} (h + e_a + e_{\mathbf{0}})|(h' + e_{\tau(a)} + e_{\tau(\mathbf{0})}).$$

Theorem. 1. Let τ be the permutation induced by an automorphism of a regular subgroup of $GA(r,2)$. Then S_τ is a transitive binary extended perfect code.

2. Transitive codes S_τ and $S_{\tau'}$ are isomorphic iff their sets of codewords of weight 4 are isomorphic.

3. For any $r \ge 3$, $r \ne 5$ there is a non-Mollard 1-neighbor transitive code S_τ of length 2^{r+1}.

Some of the above results were published in [3].

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Complemented subgroups in infinite groups

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It is well-known that if a group has a family of subgroups satisfying the same property then one can grasp several information about the structure of the whole group (see for example [3, 4]). For instance, in a group of infinite rank the family of all subgroups of infinite rank has a strong influence on the group itself as showed in [1] and [2]. Recall that a group G has *finite rank* $\rho(G) = r$ if every finitely generated subgroup of G can be generated by at most r elements and r is the least positive integer with this property. If such an integer r does not exist, then we say that G has *infinite rank*.

The property we are mainly interested in is the complementation. We say that a subgroup H of a group G is *complemented in* G if there exists a subgroup K of G such that $G = HK$ and $H \cap K = 1$. The subgroup K is called a complement of H in G. In [5] Hall proved that a finite group has every subgroup complemented if and only if it is supersoluble with elementary abelian Sylow subgroups.

The aim of this talk is to deal with some classes of infinite groups. Indeed, we will characterize two families of infinite groups in which some classes of subgroups are complemented, facing in particular the case of infinite rank groups.

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The regular two-graph on 276 vertices revisited

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The McLaughlin graph Γ is the unique strongly regular graph with parameters $(275, 162, 106, 81)$. Its automorphism group contains the sporadic finite simple group McL (see [3]) as a subgroup of index 2. The switching class of the graph $\Delta = K_1 \cup \Gamma$ is a regular two-graph on 276 vertices, whose uniqueness was shown by Goethals and Seidel [1].

The vertices of the graph Δ can be represented by a set of norm 3 vectors in $\mathbb{R}^{24}$, where two vertices are adjacent (resp. non-adjacent) if and only if the corresponding inner product is 1 (resp. 0). If we augment the lattice generated by these vectors by adding a root r in such a way that all vertices lie in the hyperplane $\{x \in \mathbb{R}^{24} \mid (r, x) = 1\}$, then this lattice L not only represents Δ but all graphs in its switching class.

While it is known that there are a number of strongly regular graphs with parameters $(276, 140, 58, 84)$ contained in this switching class (see [2, 4]), we would like to show that some graphs have a distinguished property which none of the regular graphs in the switching class has. The remarkable property we noticed is that the sublattice generated by the vertices of a graph is a proper sublattice of L. Except the graph Δ, if a graph in the switching class generates a proper sublattice of L, then it is of index 2. Our classification reveals that there are exactly four such graphs, none of which is regular.

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Ramification structures for quotients of the Grigorchuk groups

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This is joint work with A. Thillaisundaram

Groups of automorphisms of regular rooted trees have been studied for years as an important source of groups with interesting properties. For example, the Grigorchuk groups provide a family of groups with intermediate word growth and the torsion Grigorchuk groups constitute a counterexample to the General Burnside Problem. For these groups there is a natural family of normal subgroups of finite index, which are the level stabilizers. The goal of this talk is to show that the quotients by such subgroups admit a ramification structure. Roughly speaking, groups of surfaces isogenous to a higher product of curves are characterised by the existence of a ramification structure. Recall that an algebraic surface S is *isogenous to a higher product of curves* if it is isomorphic to $(C_1 \times C_2)/G$, where C_1 and C_2 are curves of genus at least 2, and G is a finite group acting freely on $C_1 \times C_2$.

In this talk, we first introduce the Grigorchuk groups and then we show that their quotients admit ramification structures, providing the first explicit infinite family of 3-generated finite 2-groups with ramification structures that are not Beauville.

The number of chains of subgroups for certain finite symmetric groups

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Throughout this paper, all groups are assumed to be finite. The lattice of subgroups of a given group G is the lattice $(L(G), \leq)$ where $L(G)$ is the set of all subgroups of G and the partial order $\leq$ is the set inclusion. The study of chains of subgroups in this paper describes the set of all chains of subgroups of G that end in G which is used to solve many computational problems in fuzzy group theory. Tărnăuceanu, and Bentea [5] gave an explicit formula for the number of chains of subgroups in the lattice of a finite cyclic group by finding its generating function of one variable. The problem of counting chains of subgroups of a given group G has received attention by researchers with related to classifying fuzzy subgroups of G under a certain type of equivalence relation (see [1] [2], [3], [4] , [6]). In this paper, we determined the number of chains of subgroups in the subgroup lattice of a certain finite symmetric groups using computational technique induced by isomorphism classes of subgroups of G with respect to this natural equivalence relation. This work also showed that fuzzy subgroup is simply a chain of subgroups in the lattice of subgroups.

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From soluble to π-separable groups

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All groups considered are finite. We present an extension of the theory of finite soluble groups to the universe of π-separable groups, π a set of primes, i.e. finite groups each of whose composition factors is either a π-group or a π'-group, where π' stands for the complement of π in the set of all prime numbers. Classical results of Hall theory state that soluble groups are characterized by the existence of Hall ρ-subgroups for all sets of primes ρ (P. Hall [5, 6]). When the set of primes π is fixed, π-separable groups have Hall π-subgroups, and also every π-subgroup is contained in a conjugate of any Hall π-subgroup (S. A. Čunihin [3]). We analyze the reach of π-separability further from soluble groups, by means of complement and Sylow bases and Hall systems, based on this remarkable property of π-separable groups. We also show that π-separable groups have a conjugacy class of subgroups which specialize to Carter subgroups, i.e. self-normalizing nilpotent subgroups, or equivalently, nilpotent projectors, when specializing to soluble groups. These Carter-like subgroups enable an extension of the existence and conjugacy of injectors, associated to Fitting classes with adequate stronger closure properties, to π-separable groups, in the spirit of the role of Carter subgroups in the theory of soluble groups.

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Automorphisms of affine surfaces and the Thompson group T

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We describe the correspondence between the unit circle and the divisor at infinity for some affine algebraic surfaces, which induces the correspondence between the Thompson group T and the automorphism groups of considered surfaces. In particular, we introduce surfaces of Markov type and describe their automorphism groups. Finally, we raise the question about integer point orbits on these surfaces. The answer to it might be useful in studying the Uniqueness conjecture for Markov numbers.

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On the representation of lattices by subgroup lattices of locally finite 2-groups

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The problem of representation of lattices by subgroup lattices occupies a prominent place in the aspect of representation of lattices by derived lattices of one kind or another. One of the most striking result here is due to Ph. M. Whitman [2] and states that every lattice can be embedded in the subgroup lattice of some group. For a given class **K** of groups, we say that **K** is *lattice-universal* if every lattice is embeddable in the subgroup lattice of some group from **K**. In this sense the class of all groups is lattice-universal.

The author proved in [1] that the class of free Burnside groups satisfying the identity $x^k = 1$, where k is odd and $k \geq 665$, is also lattice universal. In connection with this result the following question arises: *which are natural classes of locally finite groups satisfying the condition of lattice universality?* The following theorem gives us one of such important classes.

A lattice is called *algebraic* if it is complete and its every element is a join of compact elements.

If a lattice L is complete and a subset M of L has the property that $\bigvee S,\ \bigwedge S \in M$ for every nonempty subset $S \subseteq M$, then M is called *a complete sublattice* of L. It is well known that a complete sublattice of the subgroup lattice of a any group is algebraic.

Theorem. *Let* **K** *be an abstract class of groups satisfying the following conditions:*

(1) **K** *contains a two-element group;*

(2) **K** *is closed under restricted direct products, semidirect products and direct limits over totally ordered sets.*

Then every algebraic lattice is isomorphic to a complete sublattice in the subgroup lattice of some group in **K**.

It should be noted that the class of all locally finite 2-groups satisfies conditions (1), (2) of the theorem. So we get the following corollary.

Corollary 1. *Every algebraic lattice is isomorphic to a complete sublattice in the subgroup lattice of a suitable locally finite 2-group.*

Since every lattice can be embedded in some algebraic lattice, namely, in the lattice of its ideals, we obtain the following statement.

Corollary 2. *The class of all locally finite 2-groups is lattice-universal.*

In particular, from here we obtain that the class of all locally nilpotent torsion groups is lattice-universal.

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On enumeration of finite topologies

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The work continues the study of the numbers $W(p_1,\ldots,p_k)$, defined in [1] and appearing in [2–4]. By $\mathcal{V}_0(X)$ we denote the collection of all partial orders, defined on the set $X \doteq \{1,\ldots,n\},\ n\in\mathbb{N}$. There is a one-to-one correspondence between the set $\mathcal{V}_0(X)$ and the set $\mathcal{V}_0^0(X)$ of all labeled transitive digraphs, defined on X; in addition, there is a one-to-one correspondence between $\mathcal{V}_0(X)$ and the set $\mathcal{T}_0(X)$ of all labeled T_0-topologies, defined on X. Let $T_0(n) \doteq \operatorname{card}\mathcal{T}_0(X)$.

Obviously, $T_0(n) = \operatorname{card}\mathcal{V}_0(X) = \operatorname{card}\mathcal{V}_0^0(X)$. In [1] are proved a formula

$$T_0(n) \ = \sum_{p_1+\ldots+p_k=\,n} (-1)^{n-k}\,\frac{n!}{p_1!\ldots p_k!}\,W(p_1,\ldots,p_k),$$

reducing the counting of $T_0(n)$ to the calculation of numbers $W(p_1,\ldots,p_k)$ of partial orders of a special kind. Summation is performed over all partitions $(p_1,\ldots,p_k)$ of the number n into natural terms.

In [1–3] received three families of relationship equations between individual numbers $W(p_1,\ldots,p_k)$. So, if D_k is the dihedral group generated by the permutations $\left(\begin{smallmatrix}1&2&\ldots&k{-}1&k\\2&3&\ldots&k&1\end{smallmatrix}\right)$, $\left(\begin{smallmatrix}1&2&\ldots&k{-}1&k\\k&k{-}1&\ldots&2&1\end{smallmatrix}\right)$, then according to [1] we have equalities

$$W(p_{\pi(1)},\ldots,p_{\pi(k)}) = W(p_1,\ldots,p_k), \qquad \pi\in D_k. \tag{1}$$

In [2] and [3] the recurrence formulas are proved accordingly (for integer $p\geqslant 0,\ q\geqslant 0,\ m\geqslant 1$):

$$\sum_{q_1+\ldots+q_r=m}(-1)^{m-r}\,\frac{m!}{q_1!\ldots q_r!}\,W(p+1,q_1,\ldots,q_r) = \sum_{q_1+\ldots+q_r=m}(-1)^{m-r}\,\frac{m!}{q_1!\ldots q_r!}\,(r+1)\,W(p,q_1,\ldots,q_r), \tag{2}$$

$$\sum_{q_1+\ldots+q_r=\,m+1}(-1)^{m+1-r}\,\frac{m!}{(q_1-1)!\,q_2!\ldots q_r!}\,W(p,q,q_1,\ldots,q_r) = \sum_{k=0}^{p}\binom{p}{k}\,2^{\,(p-k)q}\,\sum_{l=0}^{q}\binom{q}{l}\sum_{q_1+\ldots+q_r=\,m}(-1)^{m-r}\,\frac{m!}{q_1!\ldots q_r!}\,W(k,l,q_1,\ldots,q_r). \tag{3}$$

Relations allow calculating the numbers $T_0(n)$ for all $n\leqslant 8$ (without using a computer, only by solving system (1)–(3)). For calculating the number $T_0(9)$ these relations are not enough.

In [5] five new relationship equations are obtained between individual numbers $W(p_1,\ldots,p_k)$.

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Skeleton groups and their isomorphism problem

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This is a joint work with A/Prof. Heiko Dietrich, Monash University, Australia

Classification of groups up to isomorphism is one of the main themes in group theory. A particular challenge is to understand finite p-groups, that is, groups of p-power order for a prime p. Since a classification of p-groups by order seems out of reach [1] for large exponents n, other invariants of groups have been used to attempt a classification; a particularly intriguing invariant is coclass which was introduced in 1980 by Leedham-Green and Newman [2]. Since then *coclass theory* has delivered significant insight into the structure of the p-groups of a fixed coclass.

A finite p-group of order p^n and nilpotency class c has *coclass* $r = n - c$. Recent work in coclass theory is often concerned with the study of the *coclass graph* $\mathcal{G}(p, r)$ associated with the finite p-groups of coclass r. The vertices of the coclass graph $\mathcal{G}(p, r)$ are (isomorphism type representatives of) the finite p-groups of coclass r, and there is an edge $G \to H$ if and only if G is isomorphic to $H/\gamma(H)$ where $\gamma(H)$ is the last non-trivial term of the lower central series of H. It is known [3, 4] that one of the feasible approaches for investigating $\mathcal{G}(p, r)$ is to first focus on so-called *skeleton groups*. This is mainly because of the fact [5] that almost every group in the graph $\mathcal{G}(p, r)$ is *close* to a skeleton group and thus skeleton groups determine the general structure of coclass graphs.

We introduced a systematic treatment of the skeleton groups of any coclass in [5] based on 'constructible groups' given by Leedham-Green [6]. For odd p, let S be an infinite pro-p group and informally skeleton groups can be described as twisted finite quotients of S, where the *twisting* is induced by some suitable homomorphism. The first major step to study these groups is to investigate when two skeleton groups are isomorphic. An isomorphism between skeleton groups induced by the automorphism of S is called an *orbit isomorphism* in [4]; this situation is particularly interesting since it means that the construction of skeleton groups up to isomorphism depends solely on the structure of S. However, it is also shown in [4] that there exist *exceptional isomorphisms* between skeleton groups which are not induced by automorphisms of S. In [5] we investigate when all isomorphisms between skeleton groups can be realised by orbit isomorphisms. Here we present some general results of this favour including two interesting special cases where we can completely solve the isomorphism problem of skeleton groups.

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Bipartite-threshold graphs

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A triple of distinct vertices (x, v, y) in a graph $G = (V, E)$ such that $xv \in E$ and $vy \notin E$ is called *lifting* if $\deg(x) \leq \deg(y)$ and *lowering* if $\deg(x) \geq 2 + \deg(y)$ [1]. A transformation φ of a graph G that replaces G with $\varphi(G) = G - xv + vy$ is called an edge rotation corresponding to a triple of vertices (x, v, y).

For a lifting (lowering) triple (x, v, y), the corresponding edge rotation is called *lifting (lowering)*. An edge rotation in a graph G is lifting if and only if its inverse in the graph $\varphi(G)$ is lowering.

For bipartite graph $H = (V_1, E, V_2)$, let $\mathrm{dpt}H(V_1)$ and $\mathrm{dpt}H(V_2)$ denoted degree partitions of parts V_1 and V_2.

Let $G = (K(V_1), E, V_2)$ be a threshold graph [2], where $K(V_1)$ is a complete subgraph and $H = (V_1, E, V_2)$ is a bipartite subgraph of G. H is called the *sandwich-subgraph* of G.

Note that every threshold graph has no lifting triples and every graph can be obtained by a sequence of lowering rotations of edges from some threshold graph [1].

A bipartite graph $H = (V_1, E, V_2)$ is called a *bipartite-threshold graph* if it has no lifting triples such that 1) $x, y \in V_1$ and $v \in V_2$ or 2) $x, y \in V_2$ and $v \in V_1$.

Theorem. *Let $H = (V_1, E, V_2)$ be a bipartite graph. Then following conditions are equivalent.*
1) H is the sandwich-subgraph of a threshold graph $G = (K(V_1), E, V_2)$,
2) H is the sandwich-subgraph of a threshold graph $G = (K(V_2), E, V_1)$,
3) Neighborhoods of vertices of each parts V_1 and V_2 are nested,
4) Neighborhoods of vertices of the part V_1 are nested,
5) Neighborhoods of vertices of the part V_2 are nested,
6) H is a bipartite-threshold graph,
7) H has no lifting triples (x, v, y) such that $x, y \in V_1$ and $v \in V_2$,
8) H has no lifting triples (x, v, y) such that $x, y \in V_2$ and $v \in V_1$,
9) $\mathrm{dpt}H(V_2) = \mathrm{dpt}H(V_1)^*$,
10) $\mathrm{dpt}H(V_1) = \mathrm{dpt}H(V_2)^*$,
where * is the conjugate transformation of partitions.

For a bipartite graph $H = (V_1, E, V_2)$ and a triple of distinct vertices (x, v, y) such that 1) $x, y \in V_1$ and $v \in V_2$ or 2) $x, y \in V_2$ and $v \in V_1$, we say that corresponding edge rotation preserves parts of H.

Corollary. *Every bipartite graph can be obtained from some bipartite-threshold graph by a sequence of lowering rotations of edges which preserve parts of bipartite graphs.*

Using these results and Kohnert's criterion for a partition to be graphical [3], we give a new simple proof of the well-known Gale–Ryser theorem on representation of two partitions by degree partitions $\mathrm{dpt}H(V_1)$ and $\mathrm{dpt}H(V_2)$ of the parts of some bipartite graph $H = (V_1, E, V_2)$.

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Pseudospectrum energy of graphs

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Let G be a graph of order n and $A(G)$ be its $(0,1)$ adjacency matrix. The set of eigenvalues of $A(G)$ is said to be *the spectrum of* G. Let $\lambda_1(G) > \lambda_2(G) > \cdots > \lambda_n(G)$, the largest eigenvalue λ_1 is called *the spectral radius.* The *energy of graph* is the sum of absolute values of them (see [?]), i.e.

$$E = E(G) = \sum_{i=1}^{n} |\lambda_i|.$$

Moreover, the resolvent matrix, i.e. $R_\zeta(M) := (\zeta I - M)^{-1}$ where ζ is complex variable, M is a matrix of $n \times n$ and I is the unit matrix. $R_\zeta(M)$ has been investigated in graph theory by Ivan Gutman et al and some algebric properties were established in ([?], 2016).

The resolvent matrix of $A(G)$ is $R_\zeta(A(G)) = (\zeta I - A(G))^{-1}$ and its eigenvalues are

$$\frac{1}{\zeta - \lambda_i}, \; i = 1, 2, \ldots, n.$$

In [?], the resolvent energy of G was defined as

$$ER = ER(G) = \sum_{i=1}^{n} \Big|\frac{1}{n - \lambda_i}\Big| \tag{1}$$

where $\zeta = n$ and $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigenvalues.

To have more information about an object represented by a matrix, the pseudospectrum energy of graph G has been investigated in graph theory and some algebraic properties was established in [?]. For a given positive integer number ε and real eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$ of $A(G)$, the pseudospectrum energy of graph G is the following set:

$$PE = PE(G) = \{\sum_{i=1}^{n} \Big|\frac{1}{n - \lambda_i}\Big| > \varepsilon_j^{-1}, \quad j = 1, 2, 3, \ldots\}. \tag{2}$$

One can observe that $PE(G)$ is the set of the possible lower bounds to resolvent energy (2) and it lies between the set of the eigenvalues of $A(G)$ and $R_\zeta(A(G))$. We refer to [?] which gave a wide studies and applications to the pseudospectrum phenomena.

In this talk, we will provide some properties and examples on the pseudospectrum energy of graphs.

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The derived series of Sylow 2-subgroups of the alternating groups and minimal generating sets of their subgroups

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The Sylow 2-subgroups of symmetric groups were described by U. Dmitruk and V. Sushchanskii [1] who presented the elements of these groups as tables, i.e. the ordered sets of polynomials of the certain form.

We consider the derived series of Sylow 2-subgroups of an alternating group and research their minimal generating sets. Sylow 2-subgroups of an alternating group have been previously investigated by the author [7]. In the present investigation, we continue our research [3] about the commutator subgroup of Sylow 2-subgroups of alternating groups.

The commutator width of a group G, denoted by $cw(G)$ [2], is the maximum of commutator lengths of elements of its derived subgroup $[G,G]$. The size of minimal generating sets of the commutator of Sylow 2-subgroup $Syl_2 A_{2^k}$ of A_{2^k} is found. This investigation continues previous investigations [3–7], where minimal generating sets of Sylow 2-subgroups of alternating groups were found. Let B be a group and let C_n denote a cyclic group of order n. For a permutational wreath product with form $B \wr C_n$, we prove that the commutator width [2] is $cw(B \wr C_n) \leq \max(1, cw(B))$.

Further we prove that the commutator of $Syl_2 A_{2^k}$ has the form $Syl'_2 A_{2^k} = Syl_2 A_{2^{k-1}} \boxtimes Syl_2 A_{2^{k-1}}$, where the subdirect product is defined by k relations. The structure of this commutator for the case $k \neq 2^m$ is additionally found by us. Being more precise, we find that $Syl'_2 A_{4k} = Syl'_2 S_{2^{l_1}} \times Syl'_2 S_{2^{l_2}} \times \cdots \times Syl'_2 S_{2^{l_m}}$.

The orders and commutator width of the first, second and third commutator subgroups of $Syl_2 A_{2^k}$ are also found.

For an upper bound of $cw(B \wr C_n)$, we also prove that $cw(B \wr C_n) \leq \max(1, cw(B))$, where $n \geq 2$ and B is a group. This is a generalization of the result of Nikolov [8] who considered a wreath product with a perfect passive group which is not required to be perfect now.

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Indices not divisible by a given prime in factorised groups

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In this contribution we show that a finite group $G = AB$ which is the product of two subgroups A and B has a central Sylow p-subgroup if and only if p does not divide the conjugacy class size (also called index) of any element in $A \cup B$, where p is a fixed prime. Indeed, this result can be framed in a current research line, namely to analyse how the indices of some elements in the factors of a factorised group influence the structure of the whole group. It is to be mentioned that the classification of finite simple groups and some properties of the prime graph of (almost) simple groups are used in our approach.

Trees and cycles

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Let T be a tree on n vertices; we may regard the edge $e = \{i, j\}$ as the transposition (i, j) which swaps i and j. A classic result by Dénes [1] shows that multiplying together all the transpositions corresponding to the edges of T in any order results in an n-cycle. If T is a star then each distinct ordering gives a distinct cycle and so all possible cycles arise; otherwise some cycles are given by multiple orderings and others by none. We call the collection of precise numbers of orderings which give each cycle of T its multiplicity or frequency distribution. In this talk we detail some progress made in our investigation of frequency distributions of trees. In particular we give a formula which calculates how many distinct cycles arise for a given T, and we show that finding its distribution is equivalent to counting the linear extensions of a particular subclass of partial orders which depends on T. We then use this characterisation to find some information about the distributions of a few particular classes of tree.

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Thin Q-polynomial distance-regular graphs

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This is joint work with Jack H. Koolen, Meng-Yue Cao and Jongyook Park

In this talk, we investigate distance-regular graphs with induced subgraphs $K_{r,t}$, where $1 \leq r \leq t$ are integers. In particular, we show that if a distance-regular graph Γ contains an induced subgraph $K_{2,t}$ $(t \geq 2)$, then t is bounded above by a function in $\frac{b_1}{\theta_1+1}$, where θ_1 is the second largest eigenvalue of Γ. We then apply this bound to thin Q-polynomial distance-regular graphs with large a_1 to show that c_2 is bounded above by a function in $\frac{b_1}{\theta_1+1}$.

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On perfect 2-colorings of Hamming graphs

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This is joint work with Evgeny Bespalov, Denis Krotov, Aleksandr Matiushev, and Konstantin Vorob'ev

This talk aims to overview the progress in the characterization of perfect colorings of Hamming graphs $H(n,q)$, mainly focusing on the nonbinary case $q>2$ and on new parameters of perfect 2-colorings.

A *perfect k-coloring* (or an equitable k-partition) f of a graph G is a surjective function from the vertex set to the set of *colors* $\{1,\dots,k\}$ such that each vertex x of color i is adjacent to exactly $s_{i,j}$ vertices of color j, where $s_{i,j}$ is some constant that does not depend on the choice of x. The matrix $S=(s_{i,j})$ of order k is called the *quotient matrix* of the perfect coloring f. The quotient matrix of a 2-coloring of a regular graph is completely defined by parameters $b=s_{1,2}$ and $c=s_{2,1}$.

Given n and q, the *Hamming graph* $H(n,q)$ is the graph with the vertex set $\mathbb{Z}_q^n$ such that vertices x and y are adjacent if and only if the Hamming distance $d(x,y)$ between them is 1.

Firstly, we conditions necessary for the existence of perfect colorings of Hamming graphs with given parameters. Most of these conditions are based on algebraic and arithmetic properties of colorings, distributions of colors in faces, and distance-regularity of Hamming graphs.

Next, we describe several constructions of perfect colorings. The simplest constructions arise from coverings of Hamming graphs, MDS codes, and 1-perfect codes, while more complex constructions combine some of the previous ones and provide new admissible parameters of perfect colorings.

At the end of the talk, we summarize results on the admissibility of parameters. In particular, when q is a prime power, we find sufficient conditions for the existence of a (b,c)-coloring in $H(n,q)$ for all large enough n. Moreover, we provide tables with admissibility statuses for parameters of colorings in q-ary n-dimensional Hamming graphs for small n and $q=3$, 4, and 6.

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Algebraic and computer models of parquethedra in the processes of describing their combinatorial types and filling the space

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A parquethedron is a convex polyhedron that has regular or parquet faces. Recall that a parquet polygon is a convex polygon composed of a finite and greater number of equiangular polygons (see [1]). The problem of classification of parquethedrons with equal edges is close to the solution. They are described up to similarity (see [2]). There are ten other types of parquet polygons that cannot serve as faces of parquethedrons with equal edges. In this talk we discuss the problem of describing the combinatorial types of such bodies.

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On distributions over classes of conjugate elements and pairs of orders of products of two of them $(2 \times 2, 2)$-triples of involutions of some groups

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This is joint work with Aleksei Makosi

The triple of involutions (i_1, i_2, i_3) of a group G that generates this group and satisfies the condition $i_1 i_2 = i_2 i_1$ is called its $(2\times 2, 2)$-triple of involutions, and in this case the group G itself is called $(2\times 2, 2)$-generated (see [1]). Every $(2\times 2, 2)$-triple of involutions is characterized by the pentad (m, n, C_1, C_2, C_3), where m, n are the orders of products of non-permutable involutions $i_1 i_3$, $i_2 i_3$ and $m \leq n$, and C_k is the name of the class of conjugate involutions containing the involution i_k, for $k = 1, 2, 3$. These names are usually written as $2A, 2B, \ldots$. We do not distinguish $(2 \times 2, 2)$-triples of involutions that have the same pentads of the specified type.

Obviously, $(2 \times 2, 2)$-triples of involutions of a group, translated into each other by its automorphism, have the same pentads. The following question naturally arises: are there two $(2 \times 2, 2)$-triples with the same pentad such that any automorphism of this group does not map one triple to another?

Theorem 1 *If G is one of the following simple groups: an alternating group A_i of degree $i \leq 16$, a sporadic group M_{11}, M_{12}, M_{22}, M_{23}, M_{24}, J_1, J_2, J_3, HS, Suz, Ru, McL, or one of the linear groups: $L_2(n)$, where $n \leq 32$, $L_2(p)$ for primes p such that $37 \leq p \leq 241$; $L_3(n)$, where $n \leq 7$; $L_4(n)$, where $n \leq 5$; $L_2(49)$, then the empty sets or pentads (m, n, C_1, C_2, C_3) presented in* [2] *and only these pentads correspond to $(2 \times 2, 2)$-triples of involutions of the group G.*

The theorem contains more groups than its weaker version from [3].

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Finite factorized groups with w-supersoluble subgroups

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This is joint work with V.S. Monakhov

Throughout this paper, all groups are finite and G always denotes a finite group. The formations of all supersoluble groups and groups with abelian Sylow subgroups are denoted by $\mathfrak{U}$ and $\mathcal{A}$, respectively. The notation $Y \leq X$ means that Y is a subgroup of a group X and $\mathbb{P}$ is the set of all primes. The symbol $G^{\mathfrak{X}}$ denotes the intersection of all normal subgroups N of G with $G/N \in \mathfrak{X}$, for any non-empty formation $\mathfrak{X}$.

A. F. Vasil'ev, T. I. Vasil'eva and V. N. Tyutyanov in [1] proposed the following definitions. A subgroup H of a group G is called $\mathbb{P}$-*subnormal* in G if either $H = G$ or there is a chain subgroups

$$H = H_0 \leq H_1 \leq \ldots \leq H_n = G, \ |H_i : H_{i-1}| \in \mathbb{P}, \ \forall i.$$

A group G is called w-*supersoluble* (widely supersoluble) if every Sylow subgroup of G is $\mathbb{P}$-subnormal in G. Denote by $\mathrm{w}\mathfrak{U}$ the class of all w-supersoluble groups, see [1].

Note that $\mathfrak{U} \subset \mathrm{w}\mathfrak{U}$ and $[E_{7^2}]S_3 \in \mathrm{w}\mathfrak{U} \setminus \mathfrak{U}$. Here $[E_{7^2}]S_3$ is a minimal non-supersoluble group of order $2 \cdot 3 \cdot 7^2$. In [1, Theorem 2.7, Proposition 2.8] it is proved that $\mathrm{w}\mathfrak{U}$ is a subgroup-closed saturated formation and every group from $\mathrm{w}\mathfrak{U}$ has an ordered Sylow tower of supersoluble type.

The class $\mathrm{w}\mathfrak{U}$ is not closed under taking product of $\mathbb{P}$-subnormal subgroups. The factorizable groups $G = AB$ with w-supersoluble $\mathbb{P}$-subnormal factors A and B were investigated in many works, see, for example, references in [2], [3]. Such a group G is w-supersoluble if $G^{\mathcal{A}}$ is nilpotent [4, Theorem 4.7]. If A and B are nilpotent and $\mathbb{P}$-subnormal in $G = AB$, then G is supersoluble [2], [5]. If A is nilpotent and B is supersoluble, then G may be not w-supersoluble, see [6, Example 1]. New sufficient conditions for w-supersolubility were obtained by A. Ballester-Bolinches et al. in [3].

In the present paper, we transfer the results of work [3] on arbitrary subgroup-closed saturated formation $\mathfrak{F}$ such that $\mathfrak{U} \subseteq \mathfrak{F} \subseteq \mathrm{w}\mathfrak{U}$.

Theorem. *Let $\mathfrak{F}$ be a subgroup-closed saturated formation such that $\mathfrak{U} \subseteq \mathfrak{F} \subseteq \mathrm{w}\mathfrak{U}$. Let A and B be $\mathbb{P}$-subnormal subgroups of a group $G = AB$ such that $A \in \mathfrak{F}$ and B is nilpotent. If B permutes with each Sylow subgroup of A, then $G \in \mathfrak{F}$.*

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Iterated wreath products in product action and where they act

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It is well-known that the automorphism group of a d-adic rooted tree $\mathrm{Aut}(T)$ is isomorphic to the infinitely iterated wreath of copies of the symmetric group $\mathrm{Sym}(d)$ w.r.t. the imprimitive action. The group $\mathrm{Aut}(T)$ has several interesting subgroups, e.g. Grigorchuk and Gupta-Sidki groups. These are examples of just infinite groups, i.e. groups that are infinite and each of their proper quotients is finite.

On the other hand, there is another natural action of the wreath product, i.e. the product action, and we can consider *infinitely iterated wreath products w.r.t. the product action* (IIWPPA for short).

In this talk I will present some work in progress on abstract subgroups of IIWPPA and some structures they preserve. Finally, I will hint at how these ideas could be used to produce new examples of (hereditarily) just infinite groups.

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Competition numbers and phylogeny numbers of generalized Hamming graphs with diameter at most three

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This is joint work with Soesoe Zaw

Let D be a digraph. The competition (resp. phylogeny) graph of D is the graph sharing the same vertex set as D and two vertices are adjacent if and only if their out-neighbors (resp. close out-neighbors) have a non-empty intersection. The competition (resp. phylogeny) number $\kappa(G)$ (resp. $\phi(G)$) of a graph G is the least number of vertices to be added to G to make G a competition (resp. phylogeny) graph of an acyclic digraph. Let $\mathcal{GH}_{q_1,\ldots,q_d}$ be the generalized Hamming graph with diameter d such that $\mathcal{GH}_{q_1,\ldots,q_d}$ is the Cartesian product of d complete graphs of order $q_1, \ldots, q_d$, respectively. We determine the competition numbers and the phylogeny numbers of generalized Hamming graphs with diameters 2 and 3. Thanks to these results, we solve the problem raised by Wu et al. by proving that for any non-negative integer k there is a connected graph G such that $\phi(G) - \kappa(G) + 1 = k$.

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Third proof of a theorem on the intersection of abelian subgroups in finite groups

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Let G be a finite group, A and B subgroups of G. We define $M_G(A,B)$ as the set of minimal by the inclusion intersections of the form $A \cap B^g$, $g \in G$, and $m_G(A,B)$ as a subset of minimal by the order elements from $M_G(A,B)$. We put $Min_G(A,B) =< M_G(A,B) >$ and $min_G(A,B) =< m_G(A,B) >$. It is clear that $Min_G(A,B) \geqslant min_G(A,B)$, and also that

$$min_G(A,B) \neq 1 \Longleftrightarrow Min_G(A,B) \neq 1.$$

Theorem. *Let G be a finite group and A and B be abelian subgroups of G. Then $Min_G(A,B) \leq F(G)$.*

This theorem was proved by the author in 1994 (see [1]) with using a certain uniqueness theorem and the Baer-Suzuki theorem (see [2, Theorem 2.12]).

Later, in 2008, Isaacs in Theorem 2.18 from his remarkable book [2] modified our proof of the theorem referring only to the Baer-Suzuki theorem while retaining the main idea of the proof from [1].

In this work, we present a proof of the theorem under consideration, which refers only to Wielandt's theorem (see [2, Theorem 2.9]) which, under certain conditions, is an uniqueness theorem.

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Some results related to the open problem proposed by Professor Monakhov

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In this talk, we mainly investigated the arithmetic properties of second maximal subgroups of finite groups. Generally speaking, we investigated the problem proposed by Monakhov and developed the research of Meng and Guo by weakening the condition that solvability.

Complex Clifford group and unitary design

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This is joint work with Eiichi Bannai and Manabu Oura

Roughly speaking, unitary design is a finite subset of the unitary group which approximates the whole unitary group concerning integrals. It rises from the need in experiments and engineering related to quantum physics. Formally a unitary t-design is a subset X of the unitary group $U(d)$ such that

$$\frac{1}{|X|}\sum_{U\in X} U^{\otimes t}\otimes (U^{\dagger})^{\otimes t} = \int_{U(d)} U^{\otimes t}\otimes (U^{\dagger})^{\otimes t}\,\mathrm{d}U.$$

The complex Clifford group, which is relatively easy to implement in quantum physics, outstands as an infinite family of unitary 3-designs [3]. It is pointed out in [4] that the complex Clifford group fails gracefully to be a unitary 4-design, and projective 5-designs may come for free from projective 4-designs by the complex Clifford group.

The invariants of complex Clifford group of genus m were characterized by Runge's theorem [2]. It says that the space of polynomial invariants is spanned by the genus-m complete weight enumerators of binary doubly even self-dual codes.

We generalize Runge's theorem from polynomial invariants to complex conjugate polynomial invariants. A complex conjugate polynomial is a polynomial in variables x_f as well as their complex conjugates $\overline{x_f}$.

Theorem. [Analogue of [1, Lemma 4.9]] *The space of homogeneous complex conjugate invariants of degree (N_1, N_2) for the complex Clifford group $\mathcal{X}_m$ of genus m is spanned by the conjugate complete weight enumerators $ccwe(C(m))$, where C ranges over all binary doubly-even self-dual codes of length (N_1, N_2); This is a basis if $m+1 \geq (N_1+N_2)/2$.*

In particular the number of binary doubly-even self-dual codes of length $(4, 4)$ or $(5, 5)$ are both 4. Consequently we settle Conjecture 2 in [4].

Corollary *If an orbit of the complex Clifford group is a projective 4-design, then it is automatically a projective 5-design.*

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On the Möbius function of a finite group

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This is joint work with Martino Borello and Francesca Dalla Volta

The Möbius function $\mu_{\mathcal{P}} : \mathcal{P} \times \mathcal{P} \to \mathbb{Z}$ on a locally finite poset $\mathcal{P}$ is defined inductively by

$$\mu_{\mathcal{P}}(x, y) = 0 \text{ if } x \not\leq y, \quad \mu_{\mathcal{P}}(x, x) = 1, \quad \sum_{x \leq z \leq y} \mu_{\mathcal{P}}(z, y) = 0 \text{ if } x < y.$$

When $\mathcal{P}$ is the subgroup lattice $\mathcal{L}$ of a finite group G, the Möbius function $\mu_{\mathcal{L}}$ was used by Hall [6] to enumerate generating tuples of G. Actually, the knowledge of $\mu_{\mathcal{L}}$ has a number of combinatorial applications, in the context of enumerative problems where the Möbius inversion formula turns out to be applicable.

Particular attention has been payed to the case when G is a finite simple group; for instance, the values $\mu_{\mathcal{L}}(H, G)$ have been computed for any subgroup H when G is a group $\mathrm{PSL}(2, q)$ [4], a Suzuki group [5], a small Ree group [8]. In this talk we will show computations for the groups $\mathrm{PSU}(3, 2^{2^n})$ [9] and $\mathrm{PSL}(3, 2^p)$ with prime p [2].

For a finite group G, another poset considered in the literature is the "subgroup pattern" of G, namely the poset $\mathcal{C}$ of conjugacy classes of subgroups of G, where $[H] \leq [K]$ if H is contained in a conjugate of K. Pahlings [7] proved that, whenever G is solvable, the relation

$$\mu_{\mathcal{L}}(H, G) = [N_{G'}(H) : G' \cap H] \cdot \mu_{\mathcal{C}}(H, G)$$

holds for any subgroup H of G. This property does not hold in general, the Mathieu group M_{12} being a counterexample [1]. Yet, we show that this property holds for large classes of non-solvable groups, including all minimal non-solvable groups [3].

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Non-existence of sporadic composition factors for finite groups with a condition on their Gruenberg–Kegel graphs

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For a positive integer n, denote by $\pi(n)$ the set of prime divisors of n. Given a finite group G, write $\pi(G)$ for $\pi(|G|)$ and $\omega(G)$ for the set of element orders of G.

The Gruenberg–Kegel graph (or the prime graph) $GK(G)$ of G is a simple graph with the vertex set $\pi(G)$ in which two distinct vertices r and s are adjacent if and only if $rs \in \omega(G)$.

A group G is said to be recognizable by its Gruenberg–Kegel graph if for any finite group H the equality $GK(G) = GK(H)$ implies the isomorphism $G \cong H$.

M. Hagie [1] described finite groups with Grunberg-Kegel graphs as Gruenberg-Kegel graphs of sporadic groups. A. V. Zavarnitsine [2] proved the recognizability by Gruenberg–Kegel graph of the simple groups $G_2(7)$, ${}^2G_2(q)$ for $q = 3^{2m+1} > 3$, and J_4.

A. M. Staroletov [3] proved the following theorem: If a finite group G isospectral to a finite simple group L (i. e., $\omega(G) = \omega(L)$) contains a sporadic group S among its composition factors and L is not isomorphic to Alt_n for $n \geq 22$, then either $L \cong U_5(2)$ and $S \cong M_{11}$, or $L \cong S$.

In connection with the problem of recognizing finite groups by Gruenberg–Kegel graph, the question arose on the compositional structure of a finite group whose Gruenberg–Kegel graph coincides with Gruenberg–Kegel graph of a known finite simple group.

We consider sporadic composition factors of such groups and prove the following two theorems.

Theorem 1. Let H be a finite simple exceptional group of Lie type, G be a finite group with $GK(G) = GK(H)$, and S be a composition factor of G. Then S is not isomorphic to a sporadic simple group.

Theorem 2. Let $H = A_{n-1}(q)$ be a finite simple linear group over a field of order q, $n \geq 7$, $n \neq 8, 10$, G be a finite group with $GK(G) = GK(H)$, and S be a composition factor of G. Then S is not isomorphic to F_1 and F_2.

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A family of linearized polynomials and related linear sets

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This is joint work with Olga Polverino

Linearized polynomials appear in many different contexts, such as rank metric codes, cryptography and linear sets, and the main issue regards the characterization of the number of roots from their coefficients, see e.g. [1, 4, 5]. In this talk we will provide bounds and characterizations on the number of roots of linearized polynomials of this form

$$ax + b_0x^{q^s} + b_1x^{q^{s+n}} + b_2x^{q^{s+2n}} + \ldots + b_{t-1}x^{q^{s+n(t-1)}} \in \mathbb{F}_{q^{nt}}[x], \tag{3}$$

with $\gcd(s, n) = 1$. Then we present a family of linear sets defined by polynomials of shape (3) in the projective line $\mathrm{PG}(1, q^{nt})$ whose points have a small spectrum of possible weights, containing most of the known families of scattered linear sets. In particular, we carefully study the linear sets in $\mathrm{PG}(1, q^6)$ presented in [3] (see also [2]). These results have been proved in [6].

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